Vibrant Technology, Inc.

ME'scopeVES Application Note #8

Peak & Power Values in Auto Spectrum & PSD Measurements

INTRODUCTION

The **Auto Spectrum** shows how a signal's power is distributed in the frequency domain. The Auto Spectrum is calculated by multiplying the Fourier Spectrum (FFT) of a time waveform by its complex conjugate. The **Power Spectral Density** (**PSD)** is the Auto Spectrum normalized to a **1 Hz bandwidth**. The **PSD** is calculated by the dividing the Auto Spectrum by the frequency resolution (D¦**)**.

NOTE: This Application Note requires the **Signal Analysis** Option.

Power

ME'scopeVES has a command for displaying the power in a Trace, or the power in a band if the Band Cursor is turned ON.

In a time domain waveform, Power is calculated as,

$$\text{Power} = \frac{1}{T}\int_{t=0}^{T}\left(f(t)\right)^2 dt$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\text{Real}_i^{\ 2}\right)$$

where: **N** is the number of samples of Trace data used.

Real$_i$ is the time waveform data for the i[th] sample.

In a **Linear** (also called Root Mean Squared or RMS) frequency domain spectrum, **Power** is calculated as,

$$\text{Power} = \frac{1}{2}\sum_{i=1}^{N}\left(\text{Real}_i^{\ 2} + \text{Imag}_i^{\ 2}\right)$$

$$= \frac{1}{2}\sum_{i=1}^{N}\left(\text{Mag}_i^{\ 2}\right)$$

where **Real**$_i$ is the Real Part and **Imag**$_i$ is the Imaginary Part of the data for the i[th] sample, or **Mag**$_i$ is the magnitude of the data for the i[th] sample.

In a **Power** (also called Mean Squared or MS) frequency domain spectrum, **Power** is calculated as,

$$\text{Power} = \frac{1}{2}\sum_{i=1}^{N}\left(\text{Mag}_i\right)$$

where **Mag**$_i$ is the magnitude of the data for the i[th] sample.

In ME'scopeVES, Fourier Spectrum values can be calculated using either a two sided FFT or a one sided FFT. Two sided Fourier Spectrum values are 1/2 of the one sided values. Consequently, Power Spectrum values from a two sided FFT must be multiplied by 4 to compare them with Power Spectrum values from a one sided FFT. The Power calculation corrects for this in ME'scopeVES.

Power can also be displayed as a Linear quantity by taking the square root of its Power value. This is the Linear (or RMS) Power.

EXAMPLE #1 - Periodic Sine Wave

In this example, we will measure the power in a sine wave and in its Auto Spectrum and PSD functions. To eliminate the effects of leakage, we will synthesize a sine wave that is **periodic in the sampling window**. (*See* to Application Note #1 for details on leakage.)

- Execute **File | New | Data Block** in the ME'scopeVES window.
- Enter the following parameters into the dialog box.

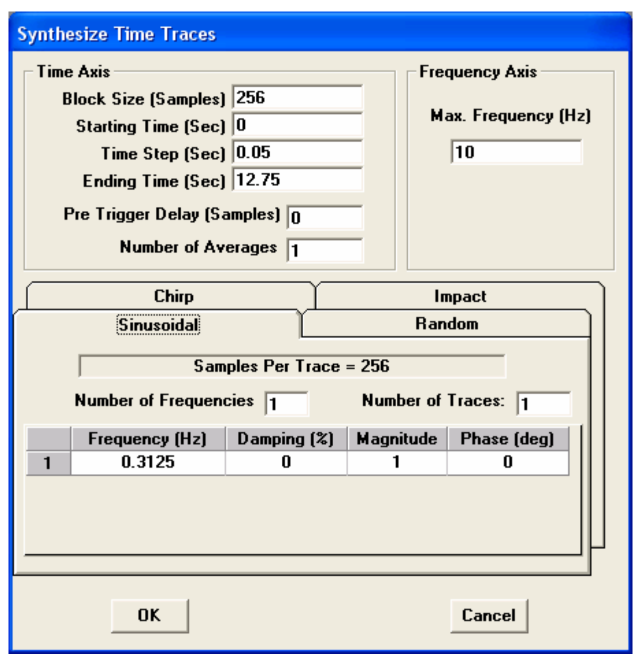

Figure 1. Dialog Box for Creating a Periodic Sine Wave.

These parameters will synthesize a **0.3125** Hz sine wave with a peak magnitude of **1.0**, and no damping.

- When all of the parameters are entered, click on **OK**. A Data Block window will open displaying the sine wave.

Notice that the Trace (shown in Figure 2) has **exactly 4 cycles** of the sine wave in it. Since an integer number of cycles are contained within the window, this signal is **periodic in the sampling window**. Since it is periodic in the window, the FFT will calculate its frequency spectrum without leakage.

Power in the Time Waveform

- Execute **Tools | Power Value**.

Notice that the power is displayed in the **Title** above the Trace. The Linear power is **0.707 g's**. Therefore, the power is **0.50 (g)^2**. Also note that the Linear Power is also displayed as a vertical bar on the right side of the Trace.

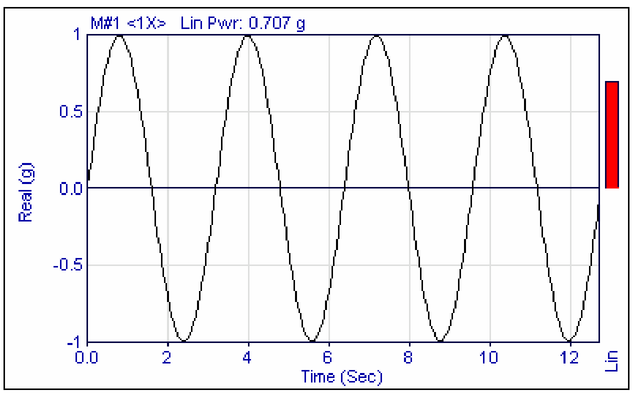

Figure 2. Power of a Periodic Sine Wave.

Calculating the Auto Spectrum

Now we will calculate the Auto Spectrum of the periodic sine wave and display its peak value and power.

- Execute **File | Options** in the Data Block window. A dialog box will open.
- On the **FFT** tab, select **Two Sided**, and click on **OK** to close the box
- Execute **Transform | Calculate | Spectra**. A dialog box will open.
- Press the **Auto Spectrum** button. The Spectrum Averaging dialog box will open.
- Click on **OK**. Another dialog box will open, allowing you to select a Data Block for storing the Auto Spectrum.
- Click on the **New File** button. Another dialog box will open. Enter "**Auto Spectrum**" and click on **OK**. The new Data Block will open.

The resulting Auto Spectrum of the sine wave has **a single peak at 0.3125 Hz**, the frequency of the sine wave.

Peak Value in the Auto Spectrum

- Use **Zoom** to display the peak clearly.
- Turn the **Line** cursor ON, and place it on the peak in the Auto Spectrum.
- **Double click** on the Y-Axis area of the graph, and change the Vertical Axis Scaling to **Linear**.

Notice that the peak value at **0.3125 Hz** is **0.25 (g)^2** or **0.5 g's,** only **half** of the original sine wave peak value. This is because **Two Sided FFT** was used to calculate the spectrum. Half of this signal (**0.5 g's**) is represented by a **negative frequency** peak at **-0.3125 Hz** and half by its **positive frequency** peak at **0.3125 Hz**.

One Sided FFT

To use the One Sided FFT and put all of the power into the positive frequency part of the spectrum,

- Execute **File | Options** in the **Periodic Sine** Data Block window. The Data Block Options box will open.
- On the **FFT** tab, select **One Sided**, and click on **OK** to close the box.
- Calculate the Auto Spectrum over again using the steps above.

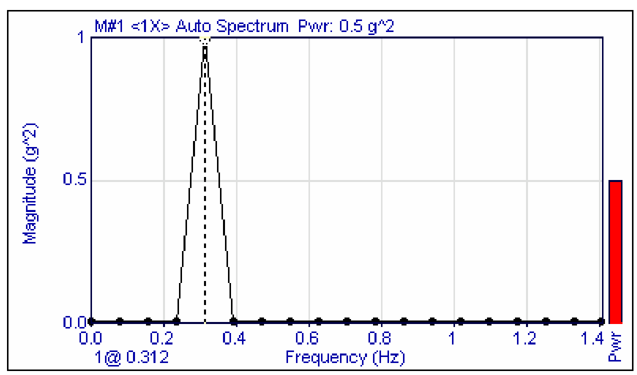

Figure 3. Auto Spectrum Using One Sided FFT.

Now, when you place the Line cursor on the peak, the cursor value will be "**1@0.3125 Hz"**.

Power in the Auto Spectrum

Notice that the Power of the Auto Spectrum is the same as the power in the time waveform, (**0.50 (g)^2** or **0.707 g's**). Hence, the power in a signal is the same in both the time & frequency domains. The power is not changed by using the FFT.

Linear Auto Spectrum

The **Linear** Auto Spectrum is the **square root** of the Power Auto Spectrum. To convert the Auto Spectrum from a **Power** to a **Linear** quantity,

- Execute **Tools | Power (MS) -> Linear (RMS)**. A dialog box will open. Click on **Yes** to modify the data.

Notice that the power (shown in Figure 4) is still **0.707 g's**, or **0.50 (g)^2**.

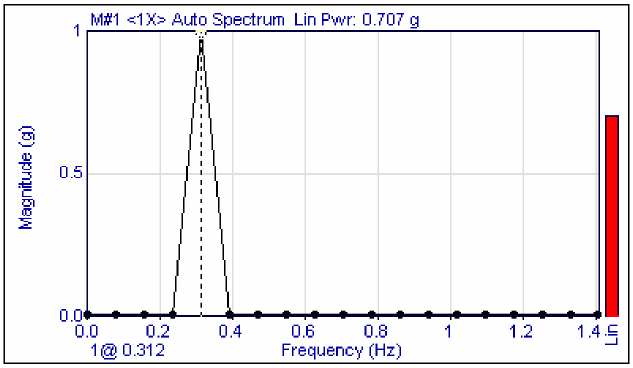

Figure 4. Linear Auto Spectrum *Showing Power=0 .707g's.*

Calculating the PSD

Now we will calculate a **PSD** for the sine wave and look at its power.

- Execute **Transform | Calculate | Spectra** in the **Periodic Sine** Data Block window. A dialog box will open.
- Choose the **Periodic Sine.BLK** and click the **PSD** button.
- The Spectrum Averaging dialog box will open. Click on **OK**. Another dialog box will open, allowing you to select a Data Block for storing the PSD.
- Click on the **New File** button. Another dialog box will open. Enter "**PSD**" and click on **OK**.

A new Data Block window with the PSD in it will open. The PSD of the sine wave will also have **a single peak at 0.3125 Hz**, the frequency of the sine wave.

- Use **Zoom** to display the peak clearly.
- Turn the **Line** cursor ON, and place it on the peak in the PSD.
- **Double click** on the Y-Axis area of the graph, and change the Vertical Axis Scaling to **Linear**.

Notice that the cursor value of the **PSD** at **0.3125 Hz** is **12.8 (g)^2/Hz**. This value is equal to the peak value of the Auto Spectrum (**1.0**) divided by the frequency increment,

D¦ = 1/T = 1/(12.8 sec) = 0.078125 Hz

To check the frequency increment,

- Execute **Edit | Properties** in the PSD Data Block window to open the Properties dialog box.

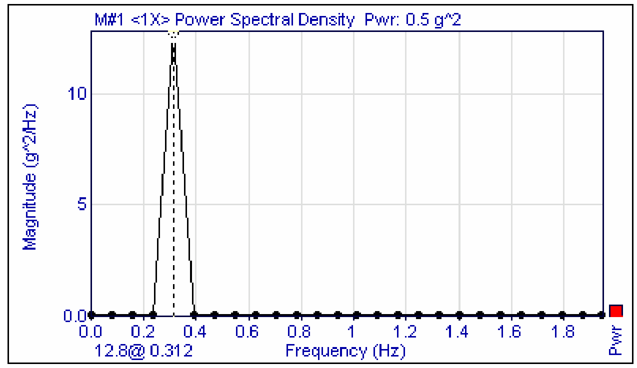

Figure 6. PSD Showing Power = 0.5 (g)^2.

Notice that the power in the PSD is **0.5 (g)^2**, as expected since it is a power quantity.

Linear PSD

The **Linear PSD** is the **square root** of the **PSD**. To obtain the Linear PSD from the PSD,

- Execute **Tools | Power (MS) -> Linear (RMS)**. A dialog box will open. Click on **Yes** to modify the data.
- Place the Line cursor on the peak in the **Linear PSD**.

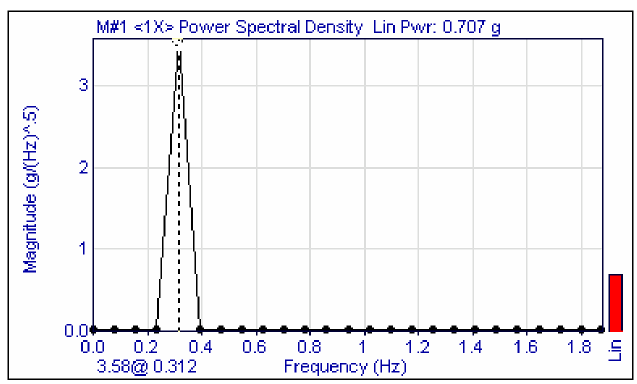

Figure 7. Linear PSD Showing Power = 0.707 *g's.*

Notice that the magnitude at 0.3125 Hz is now **3.578 (g/(Hz)$^{1/2}$)**, and that the power has remained the same; **0.707 g's** or **0.50 (g)^2**.

EXAMPLE #2 - Non-Periodic Sine Wave

Non-periodic time waveforms yield different results when transformed from the time to the frequency domain. This is due to leakage effects.

The spectrum of a non-periodic sine wave is **"smeared"** over several frequencies, instead of being non-zero only at the frequency of the sine wave. Therefore, leakage will certainly affect the peak value of a sine wave signal in the frequency domain.

To create a sine wave that is non-periodic in its sampling window,

- Execute **File | New | Data Block** in the ME'scopeVES window. The **Synthesize Time Traces** dialog box will open.
- Enter the following parameters into the dialog box.

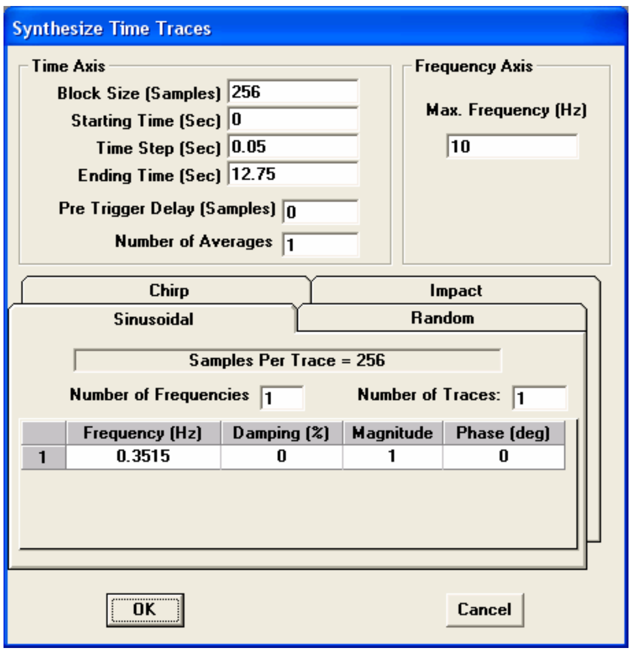

Figure 8. Dialog for Creating Non-Periodic Sine Wave.

These parameters will synthesize a **0.3515** Hz sine wave with a magnitude of **1.0**, and no damping.

- When all of the parameters are entered, click on **OK**. A Data Block window will open displaying the sine wave.

Power in the Time Waveform

- Execute **Tools | Power Value**.

Notice in Figure 9 that the power is still **0.707 g's** or **0.50 (g)^2**.

Calculating the Auto Spectrum

Now we will calculate the Auto Spectrum of the non-periodic sine wave and display its peak value and power.

- Execute **Transform | Calculate | Spectra**. A dialog box will open.
- Choose the **Non-Periodic Sine.BLK,** and click the **Auto Spectrum** button.
- The Spectrum Averaging dialog box will open. Click on **OK**. Another dialog box will open, allowing you to select a Data Block for storing the Auto Spectrum.
- Click on the **New File** button. Another dialog box will open.
- Enter "**Non-Periodic Auto Spectrum**" and click on **OK**.

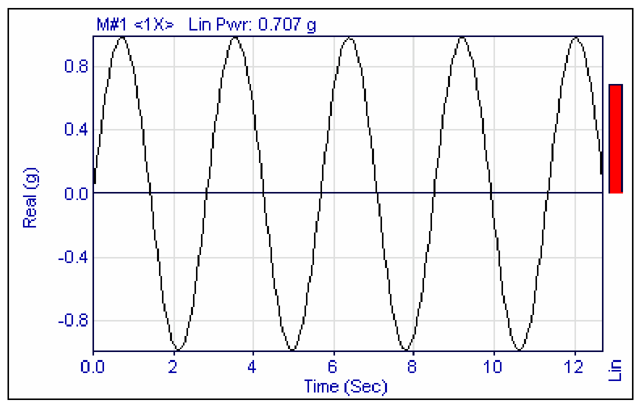

Figure 9. Non-Periodic Sine Wave.

A new Data Block window will open with the Auto Spectrum in it. The resulting Auto Spectrum of the sine wave will have a **single peak** at approximately **0.3515 Hz**, the frequency of the sine wave.

- Use **Zoom** to display the peak clearly.
- **Double click** on the Y-Axis area of the graph, and change the Vertical Axis Scaling to **Linear**.
- Turn the **Line** cursor ON, and place it on the peak in the Auto Spectrum.

Notice that the peak amplitude of the Auto Spectrum is only **0.456@0.3125 (g)^2** instead of **1.0 (g)^2**. This is due to leakage. (The true peak frequency is **0.3515** Hz, but there is no sample at that frequency.) .

To display the total power in the Auto Spectrum,

- Turn the **Line** cursor OFF.
- Execute **Tools | Power Value**

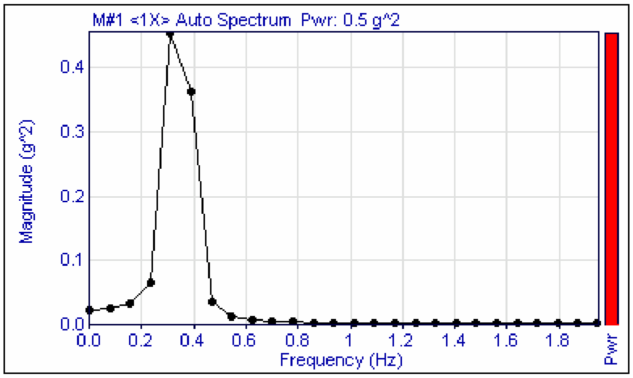

Figure 10. Auto Spectrum of a Non-Periodic Sine Wave.

Notice that the **power has remained the same** (**0.50 $(g)^2$**) in spite of the leakage. Leakage has merely spread the power over the frequency spectrum.

CONCLUSIONS

From these two examples, we can conclude the following,

1. If a sine wave is **periodic** in its sampling window and a **One Sided FFT** is used, the peak value of its Auto Spectrum is **equal** to its time domain peak value.
2. If a sine wave is **periodic** in its sampling window and a **Two Sided FFT** is used, the peak value of its Auto Spectrum is **one half** of its time domain peak value.
3. The power in the Linear (RMS) time or frequency waveform is equal to the **square root** of the power in a Power (MS) time or frequency waveform.
4. If a sine wave is **non-periodic** in its sampling window, the peak value of its Auto Spectrum or PSD will **always be less** than its time domain peak value, due to leakage.
5. The **total power** in the Auto Spectrum of a signal is the **same** as the power in its time domain signal, whether or not its time domain signal is **periodic** in the sampling window.