Vibrant Technology, Inc.

ME'scopeVES Application Note #19

Using the Z-Polynomial Method with the Stability Diagram

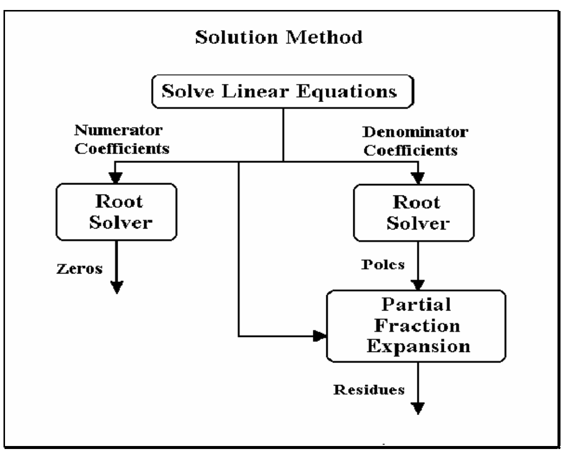
INTRODUCTION

In this Application Note you will use the new Z-Polynomial (or Z-Poly) curve fitting method to estimate modal frequencies & damping using the Stability diagram. The Z-Poly method is similar to the Orthogonal Polynomial (or Ortho-Poly) method in that it too uses the Rational Fraction Polynomial form of an FRF to curve fit the experimental data.

POLYNOMIAL CURVE FITTING

The diagram below shows the computational procedure of the two Polynomial curve fitters in ME'scopeVES. During curve fitting, sets of linear equations are solved for both the denominator and numerator polynomial coefficients of the FRF model. Then, the coefficients of the denominator polynomial are fed into a root solver which estimates the pole location, or **modal frequency & damping**, of each mode in the model.

Finally, the poles together with the numerator polynomial coefficients are used in a partial fraction expansion of the FRF model which yields a **complex residue (magnitude & phase)** for each mode.



Block Diagram of Polynomial Curve Fitting.

DETERMINING THE MODEL SIZE

The first and perhaps most difficult step of all curve fitting is to determine how many modes are represented in a set of FRF measurements. In ME'scopeVES, this can be done in two ways,

1. Counting peaks on a Mode Indicator function.
2. Using the Stability diagram.

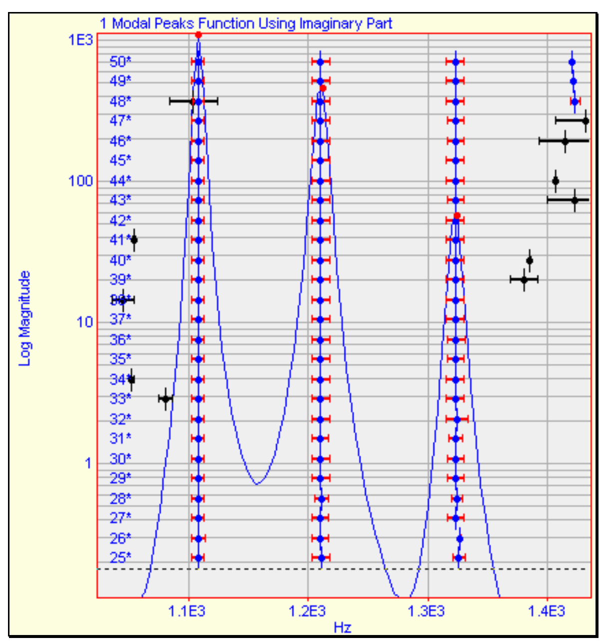
NOTE: Steps in his Application Note can be duplicated using **Visual Modal Pro** or any package that contains the **VES 450** Advanced Modal Analysis Option

ME'scopeVES has three types of Mode Indicator functions; the Modal Peaks function, Complex Mode Indicator Function (CMIF), and the Multivariate Mode Indicator Function (MMIF). Each of these indicators can be used for counting resonance peaks. Each can also be used to visually locate resonances so that more accuracy can be gained by using the frequency domain curve fitters on smaller cursor bands of data.

The Stability diagram should be used when resonance peaks cannot be clearly seen and counted. This will occur when modes are so heavily coupled together that it is not clear how many resonances are in a particular frequency band.

STABILITY DIAGRAM

The Stability diagram is a plot on many denominator polynomial solutions at once. In the diagram below, pole locations (modal frequency & damping) are displayed for model sizes from 25 to 50 modes. The function in the background is a Modal Peaks function which also shows three resonance peaks.



Stability Diagram Showing Three Stable Pole Estimates.

The Stability diagram is based on the following assumption:

"If an FRF data set contains a resonance, its pole location estimates will be the same for two or more different model sizes".

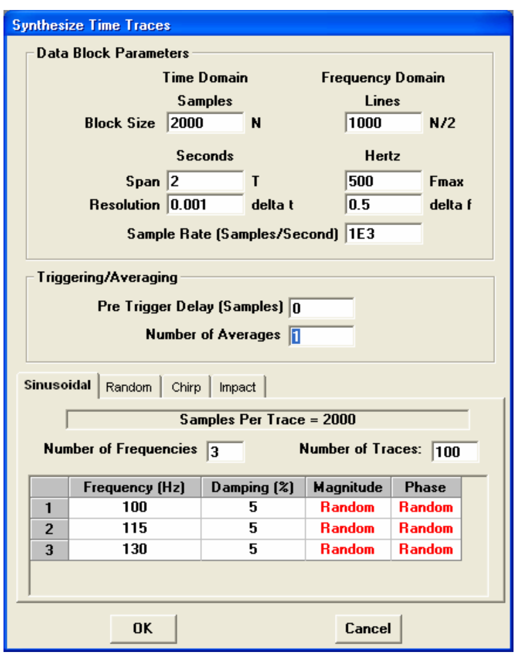
Each pole is displayed with a short vertical line at the modal frequency, and a horizontal line equal in length to the half power point modal damping, or 3 dB width of the resonance peak.

A pole estimate is said to be stable if its frequency & damping values do not change (i.e. are within user specified tolerances) from one model size to the next. In the diagram above, it is clear that three modes have stable pole estimates since the vertical lines (modal frequencies) are in alignment, and the damping lines are approximately equal in width, from one solution to the next. The other spurious pole estimates on the Stability diagram do not indicate stable pole estimates.

EXAMPLE #1

To learn how to use the Stability diagram, let's start with an example where the answer is known. We will create a set of FRFs with known pole locations in them.

- Execute **File | Project | New** in the ME'scopeVES window to start a new Project.
- Execute **File | New | Data Block** in the ME'scopeVES window. A dialog box will open, as shown below.



Dialog for the New Data Block Command.

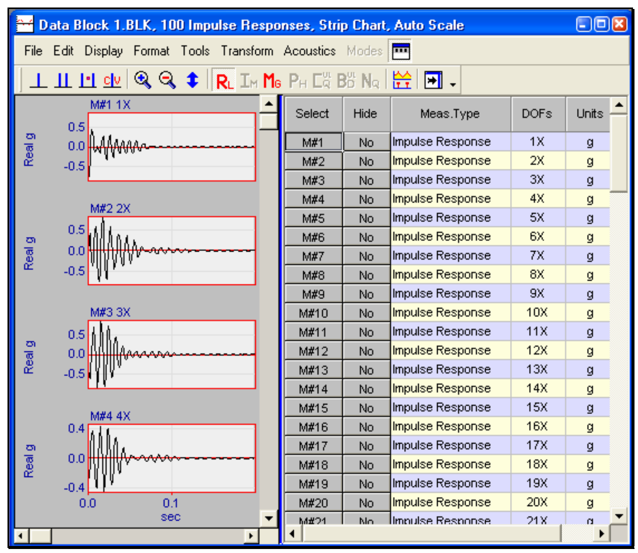
- Enter the parameters shown into the **Synthesize Time Traces** dialog box.

These parameters will create a new Data Block with 100 time domain Traces in it. Each Trace is a simulated impulse response containing three modes at 100, 115, & 130 Hz, each with 5% damping and a random residue (magnitude & phase).

Each Trace will have 2000 samples, with 0.001 seconds spacing between samples, spanning a total time of 2 seconds.

- When all of the parameters have been entered into the **Synthesize Time Traces** dialog, click on **OK**.

A Data Block window will open containing the 100 time domain Traces. Since the Fourier transform of an Impulse Response Function is an FRF, when this Data Block is FFT'd to the frequency domain, it will contain 100 FRFs, which will then be used for curve fitting.



Data Block Showing Impulse Responses.

NOTE: Your Impulse Responses may look different than those shown above because the modal Residues were randomly generated. This will not affect the frequency & damping estimates however, which we will be seeking by curve fitting the FRFs.

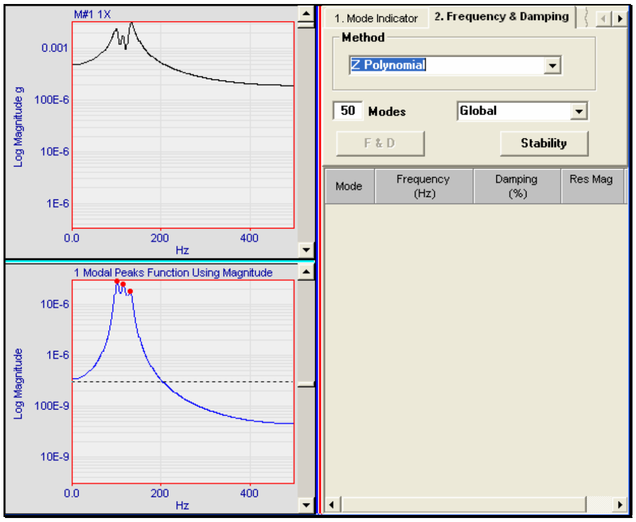
To transform the Impulse Responses to FRFs,

- Execute **Transform | FFT** in the Data Block window.

To initiate curve fitting,

- Execute **Modes | Modal Parameters** in the Data Block window.

During curve fitting, the Curve Fitting panel is displayed on the right, and the FRFs and Mode Indicator are displayed on the left, as shown below.



Data Block Window during Curve Fitting.

Mode Indicator Function

Next we will calculate a Modal Peaks function to provide a background indication of the three poles in the FRF data set.

- On the **Mode Indicator** tab, select the **Modal Peaks Function** in the **Method** section, and press the **Count Peaks** button.

A dialog box will open, allowing you to select the part of the FRFs to be summed together to form the Modal Peaks function. You always want to choose the part of the data which contains a single peak for each mode. In this case, since the Residues were randomly generated, their random phases may cause a single peak in either the Real or the Imaginary part. Therefore, the magnitude should be chosen.

- Select **Magnitude** in the dialog box, and click on **OK**.

After the Modal Peaks function has been calculated, it will be displayed below the FRFs, as shown above. The three resonance peaks are also indicated with **red dots** on the Modal Peaks function.

USING THE STABILITY DIAGRAM

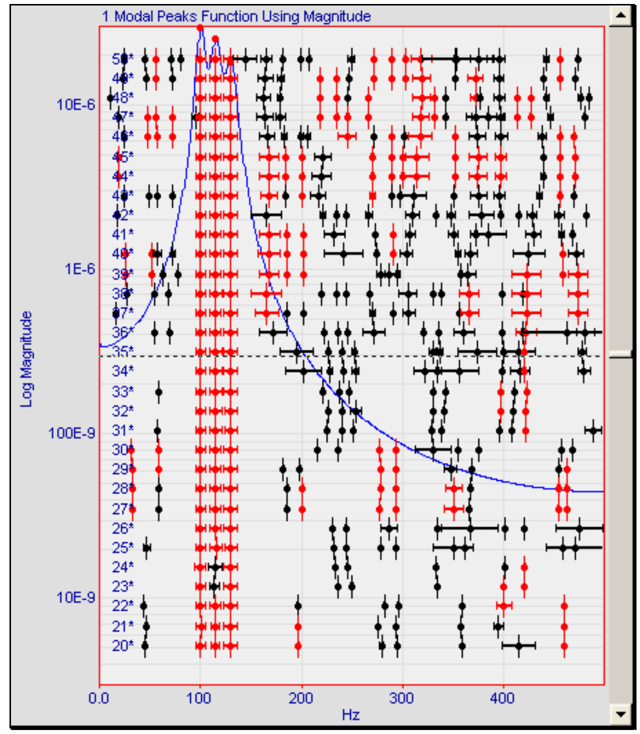
To use the Z-Polynomial method with the Stability diagram

- On the **Frequency & Damping** tab, select **Z-Polynomial** in the **Method** section.

Notice that the number in the **Modes** box is automatically changed to **50** modes. This means that the Stability diagram will display pole solutions for model sizes from 1 to 50. To calculate the solutions and display the Stability diagram,

- Press the **Stability** button on the **Frequency & Damping** tab.

After the calculation has completed, the pole estimates from all 50 solutions will be displayed, as shown below.



Stability Diagram Showing Solutions for 20 to 50 Modes.

Notice that the three modes are clearly indicated as stable poles, but there are also other spurious (computational) pole estimates on the diagram.

Maximum Damping

All of the pole estimates with heavy damping can be removed by setting the **Maximum Damping** limit on the **Curve Fitting** tab in the Data Block Options box. To open the Options dialog box with only the Curve Fitting tab showing,

- Place the mouse pointer over the **heading** in the Modal Parameters spreadsheet on the right, and click the **right** mouse button.

- In the **Stability** section on the **Curve Fitting** tab, enter a **Maximum Damping** of **5.5%.**

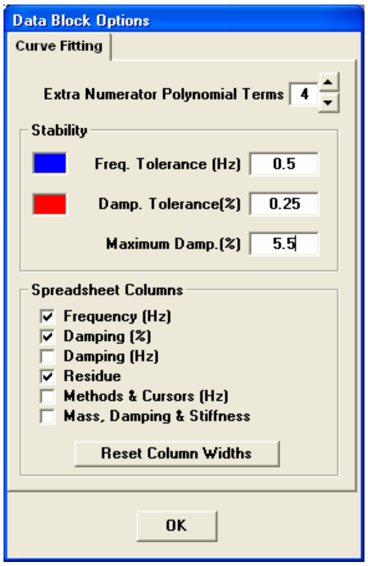
As soon as the maximum damping value is entered, the Stability diagram will be redrawn showing only pole estimates that have damping not exceeding the Maximum Damping value. In this case, 5.5% is a good number to enter since we already know that the modes of interest have damping of 5%.

With real experimental data, you won't know beforehand what Maximum Damping value to enter. However, since the value you enter automatically updates the Stability diagram, you can try different values until as many heavily damped poles as desired are removed from the diagram.

Frequency & Damping Tolerances

You can also enter frequency & damping tolerances into the Options dialog box, which will cause the pole estimates to be displayed with different colors when successive solutions are within the tolerances.

- In the **Stability** section on the **Curve Fitting** tab, enter the frequency & damping tolerances, as shown below**.**
- Click on the **color** boxes, and change them to different colors also.



Curve Fitting Tab Showing Stability Parameters.

Again, as soon as you enter the tolerances or change the colors, the Stability diagram will be redrawn to reflect your changes.

Zooming the Display

Since there is usually so much information on a Stability diagram, it is better to zoom in on some of the poles, and then scroll the zoomed display to look at others.

There are several ways to zoom the display. Depress the **Zoom** button on the Toolbar and,

- Move the mouse pointer to the left side of an area to be displayed, hold down the **left** mouse button, and drag to display a horizontal Zoom box. Then release the mouse button to display the area within the Zoom box.
- Move the mouse pointer to a spot on the graphics and click the **left** mouse to expand the display around that spot.
- Hold down the **Ctrl** key on the keyboard, move the mouse pointer to a spot, and repeatedly click the **left** mouse button to zoom around the spot, or click the **right** mouse button to zoom out (or **mooZ**) the display.

To recover the full display,

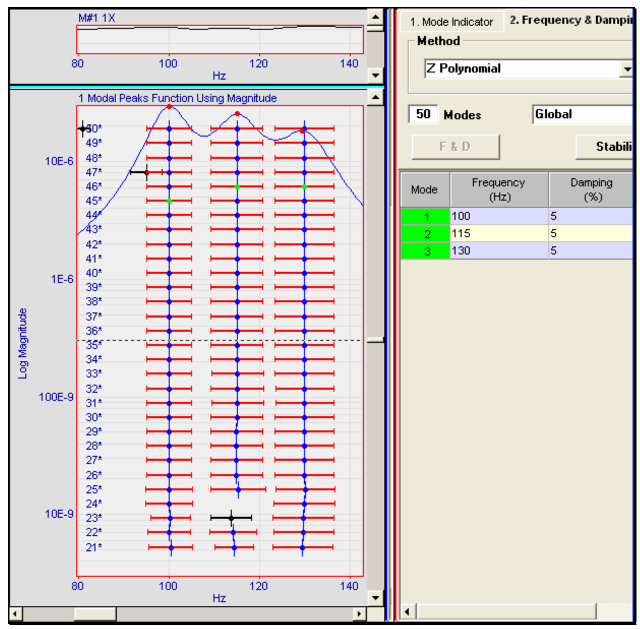
- Press the **mooZ** button on the Toolbar**.**

Selecting Estimates from the Stability Diagram

The purpose of the Stability diagram is not only to indicate where stable poles are located, but also to provide valid frequency & damping estimates for each mode.

Estimates are selected from the Stability diagram and put into the Modal Parameters spreadsheet by moving the mouse pointer near a pole and clicking the **right** mouse button. However, before selecting a pole, it is useful to preview its frequency & damping estimates.

- Zoom in around the three stable poles on the Stability diagram to display them as shown below.



Zoomed Stability Diagram Showing Stable Poles.

To display the frequency & damping next to a pole,

- Hold down either the **Shift** or **Ctrl** key, and move the cursor near to a pole on the Stability diagram.

Several pole estimates for each mode in the Stability diagram will yield **exactly** the frequency & damping values that were used to synthesize the original Impulse Responses. (This is expected when the FRF data in linear and noise free.)

- Click the **right** mouse button to select the frequency & damping estimates of a pole and place them into the Modal Parameters spreadsheet

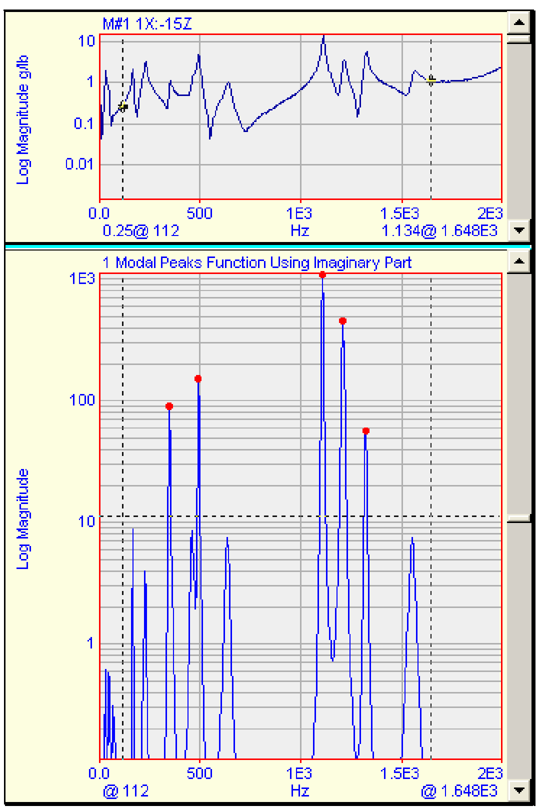
EXAMPLE #2

In this example, we will use Z-Poly method and Stability diagram to identify the frequency & damping of the modes in a set of experimental FRFs taken from the Jim Beam structure. This data is contained in the **Frequency-Based ODS** demonstration Project. To open the Project,

- Click on the **Frequency-Based ODS Demo** button on the ME'scopeVES window Toolbar.
- Stop the animation, and execute **Modes | Modal Parameters** in the Data Block window.
- On the **Mode Indicator** tab, press the **Count Peaks** button. A dialog box will open. Click on **OK**.

The Modal Peaks function will display a number of resonance peaks, including some near DC (zero frequency), which are not flexible body modes.

- Turn ON the **band cursor** and surround the flexible body modes as shown below.



Modal Peaks Function with Band Cursor ON.

Stability Diagram

To use the Z-Polynomial method with the Stability diagram,

- On the **Frequency & Damping** tab, select **Z-Polynomial** in the **Method** section, and press the **Stability** button.

Maximum Damping and Tolerances

To remove heavily damped (computational) modes from the Stability diagram,

- Open the Data Block **Options** box and enter a **Maximum Damping** value of **4%** on the **Curve Fitting** tab.
- Enter a **Frequency Tolerance** of **1 Hz** and a **Damping Tolerance** of **2%**.
- Zoom in to display the stable pole estimates more clearly.

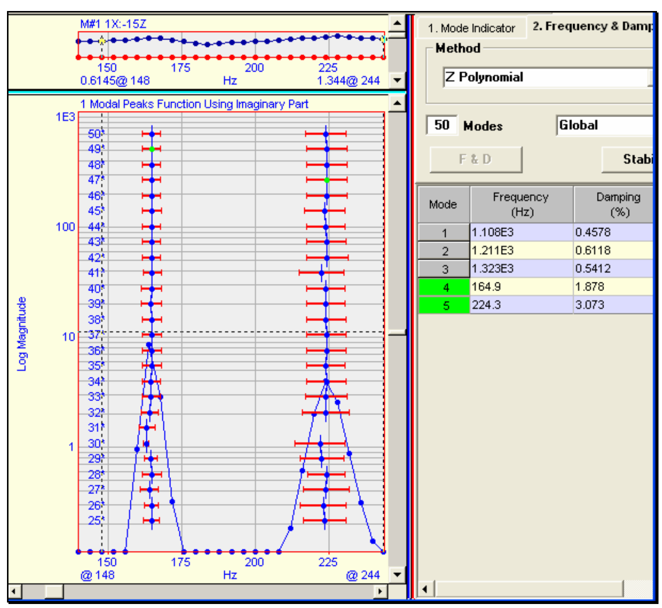
The **1108**, **1211**, and **1320 Hz** poles show good stability.

- **Right** click on a stable estimate for each of these three poles to place it into the Modal Parameters spreadsheet.

Stability Diagram in Cursor Bands

To calculate improved Stability diagrams for the rest of the modes, we will surround a few modes at a time with the band cursors, and calculate a new Stability diagram.

- Turn ON the band cursor and surround the modes at **164** and **224 Hz.**
- Press the **Stability** button again. Stable poles should now be displayed for these two modes, as shown below.
- **Right** click on a stable pole for each mode to add it to the Modal Parameters spreadsheet.



Stable Poles for the 164 and 224 Hz Modes.

NOTE: Stable poles can be displayed quickly by surrounding **one or two resonance peaks at a time** with the band cursor, and pressing the **Stability** button.

- Calculate a Stability diagram for each of the 5 remaining modes at **347, 460, 492, 635,** and **1557 Hz**.

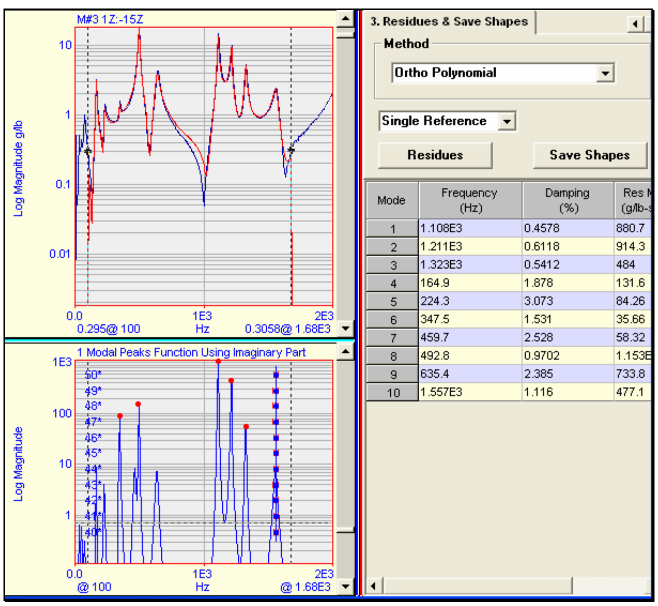
After **right** clicking on each stable pole in the Stability diagram to save it in the Modal Parameters spreadsheet, you should have a total of **10** modes in the spreadsheet, as shown below.

Estimating Residues

Once the Modal Parameter spreadsheet contains frequency & damping estimates for all modes of interest, modal residues (magnitudes & phases) are estimated by a second curve fitting step.

- Unselect all Modes in the spreadsheet by **double clicking** on the **Modes** column heading.
- Surround all **10** resonance peaks with the **band cursor.**
- On the **Residues & Save Shapes** tab, select **Ortho Polynomial** in the **Method** section, and press the **Residues** button.

When the residue curve fitting is completed, the residue estimates will be added to the Modal Parameters spreadsheet, and a **red fit function** will be overlaid on each FRF, as shown below**.**



Results after Residue Curve Fitting of 10 Modes.

Mode Shapes

Curve fitting is now complete and the parameter estimates can be saved into a Shape Table and displayed in animation on the 3D model of the Jim Beam.

The modes are not sorted by ascending frequency since they we selected in random order from the Stability diagram.

- To sort the modes, execute **Curve Fit | Sort Modes by Frequency**.

To save the modal parameters into a Shape Table,

- Press the **Save Shapes** button on the **Residues & Save Shapes** tab. A dialog box will open.
- Press the **New File** button to create a new file for the mode shapes, and click on **OK**.

After the dialogs have been closed, the new Shape Table will open with the 10 mode shapes in it. To animate the mode shapes on the model in the Structure window.

- Select the new Shape Table in the **Animation Source** drop down box on the Structure window Toolbar.
- Press the **Draw | Animate** button on the Toolbar.
- Click on the **Select** button of a mode in the Shape Table to display its shape in animation.

CONCLUSIONS

Like the Complex Exponential method, the new Z-Polynomial curve fitting method in ME'scopeVES uses the Z-transform in place of the normal frequency variable. Using the Z-transform substantially improves the numerical stability of the Rational Fraction Polynomial method so that it works well with large model sizes, thus making it ideal for use with the Stability diagram.

The Z-Poly method also executes very quickly, making it ideal for exploring multiple frequency bands of data to find stable pole estimates.

In Example #1, we found that the Z-Poly method is very accurate when applied to linear and noise free FRF data that was derived by Fourier transforming synthesized Impulse Response functions. In fact, it precisely estimated the modal parameters that were used to synthesize 100 Impulse Responses.

In Example #2, we used the Z-Poly method and Stability diagram to identify the first 10 modes of the Jim Beam structure from 99 experimental FRFs. This resulted in 3-dimensional mode shapes at 33 points on the structure.

Although it was used in two examples where other curve fitting methods might also work, the unique advantage of the Z-Poly method combined with the Stability diagram is that it can identify modes in data where counting resonance peaks may not be possible.