Vibrant Technology, Inc.

ME'scopeVES Application Note #21

Calculating Responses of MIMO Systems to Multiple Forces

INTRODUCTION

Driving forces and response motions of a vibrating structure are related in a very straightforward manner when the motions and forces are described in the Frequency Domain. The motions at **N** DOFs (points and directions) on the structure are related to forces applied to **M** DOFs by the following matrix of **NM** Frequency Response Functions (FRFs). Specifically:

$$\{X(f)\}_{Nx1} = [H(f)]_{NxM} \cdot \{F(f)\}_{Mx1} \qquad (1)$$

ME'scopeVES contains commands for investigating all aspects of the Multiple-Input Multiple Output (MIMO) relationship of equation (1). You can:

1. **Calculate all the FRFs** in the matrix from measured Forces and Responses. See Application Note #**20**.

2. **Calculate multiple Responses**, given a known matrix of FRFs and a vector of Forces. See also Application Note #**22**.

3. **Calculate multiple Forces**, given a known matrix of FRFs and a vector of Response motions.

In other words, *provide any two elements of equation (1) and the third can be calculated using MIMO commands.*

The FRF matrix can either be provided from a *Data Block* of FRF measurements, or synthesized from a *Shape Table* containing the structure's mode shapes. Both of these cases are covered in this note.

In this note, we will focus on calculating the *Responses*, given a known FRF matrix and a Force vector. Responses will be calculated as both Time Waveforms and PSDs, and compared to the measured results.

Application Note #**22** discusses use of a special Command when the excitation Forces are *sine waves*.

Steps in the application note can be duplicated using **VT-550** Visual Modal Pro or any package that includes option **VES-350** Advanced Signal Processing.

EXAMPLE DATA

This Application Note requires the FRFs that are calculated in Application Note #20. They are also provided in the More Examples sub-directory on your Installation CD.

Open **ME'scopeVES**.


Execute: **File | Project | Open**.

- Select **My Z24 Bridge.PRJ** from the **Other Examples** subdirectory.

This will open the **Z24 FRFs & Cohs.BLK** Data Block containing *Frequency Response Functions* (FRFs), *Multiple Coherence Functions* and *Partial Coherence Functions* calculated for the Z24 bridge. The project also contains the Structure file, **Z24 Bridge.STR**, the Shape Table file, **Z24 Bridge 8-mode fit.SHP**, and the Data Block file, **Z24 Bridge 2 Shaker Test Time Data.BLK**, containing the measured time domain signals.

Z24 Bridge viewed from Bern-to-Zurich highway A1.

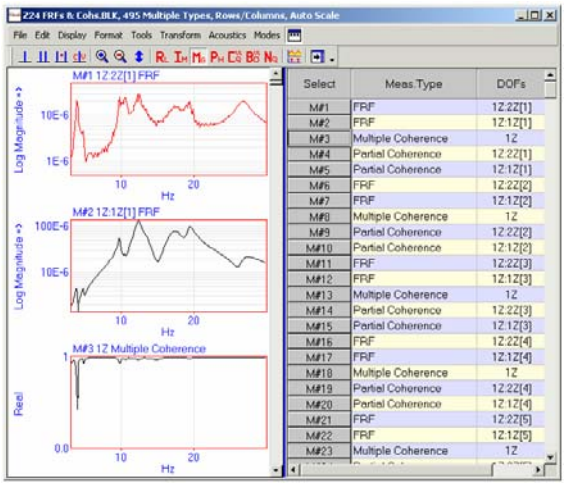
Z24 FRFs & Cohs.BLK contains **495** Traces in nine Measurement Sets. The Traces include:

198 FRFs - 144 *unique* FRFs and 6 *redundant* FRFs measured 9 times each (once per Measurement Set).

99 Multiple Coherences – one for each Response.

198 Partial Coherences– two for each Response.

REDUNDANT FRFs

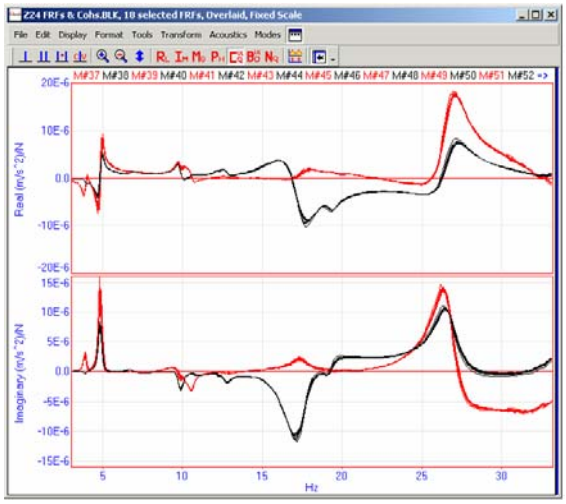


2 FRFs and Multiple Coherence for Response DOF ***1Z****.*

The FRFs from this test describe Response motions at **75** DOFs due to **2** simultaneously applied forces. Hence, the structural dynamics between the 2 forces and 75 responses is completely described by **150** FRFs.

However, **Z24 FRFs & Cohs.BLK** contains **198** FRFs. While **144** of the FRFs are *unique*, there are nine *redundant* estimates of each FRF with DOFs **1Z:1Z**, **1Z:2Z**, **2Z:1Z**, **2Z:2Z**, **-2Y:1Z** and **-2Y:2Z**.

In Application Note #**20**, we determined that the Measurement Sets contained *consistent* estimates for FRFs **1Z:1Z**, **1Z:2Z**, **2Z:1Z** and **2Z:2Z** by overlaying them. The following plot is an overlay of CoQuad plots (Real and Imaginary parts) of the nine **-2Y:1Z** (black) and **-2Y:2Z** (red) FRF estimates. It also clearly shows that these FRFs are consistent.



Nine ***-2Y:1Z*** *and* ***–2Y:2Z*** *FRF measurements overlaid.*

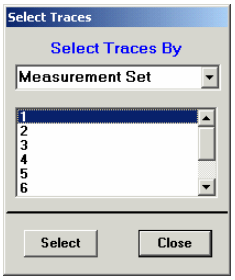
Note: the MIMO Commands use only the *first* FRF for a given DOF-pair encountered in the Traces Spreadsheet. *All other occurrences of the same FRF are ignored.*

CALCULATING MIMO RESPONSES

Responses due to the two measured force signals **:1Z[1]** and **:2Z[1]** will be calculated and compared with the **15** measured responses of Measurement Set [1]. We will do this using the FRFs in **Z24 FRFs & Cohs.BLK** and then repeat the exercise using the Mode Shapes in **Z24 Bridge 8-mode fit.SHP**.

Start by selecting the **Measurement Set [1]** FRFs in the **Z24 FRFs & Cohs.BLK** Data Block:

Execute: **Edit | Select Traces | By**. The Select traces dialog will open.



- Select **Measurement Set** from the **Select Traces By** list and click on **1** in the list below it.
- Press **Select**. (Note that **60** Traces are selected, 30 FRFs and 30 Partial Coherences.)
- Press **Close** to exit the dialog.

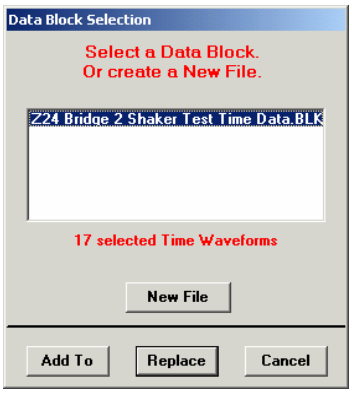
The FRFs are ready for the MIMO Response calculation. We will now copy Measurement Set [1] Time Waveforms into a new Data Block file. This will make graphic comparison of our calculated results with the measured time waveforms easier.

- Double-click on **Z24 Bridge 2 Shaker Test Time Data.BLK** in the upper pane of the Project Panel to open it.

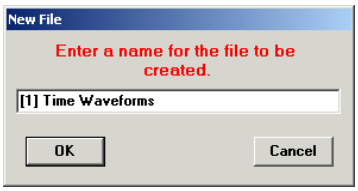
In the **Z24 Bridge 2 Shaker Test Time Data.BLK** window:

Execute: **Edit | Select Traces | By**. Select **Measurement Set [1]**. Note that **17** Traces (**2** Forces and **15** Responses) are selected.

Execute: **Copy | Traces**. The **Data Block Selection** dialog will open.



- Press the **New File** button. The **New File** dialog will open.



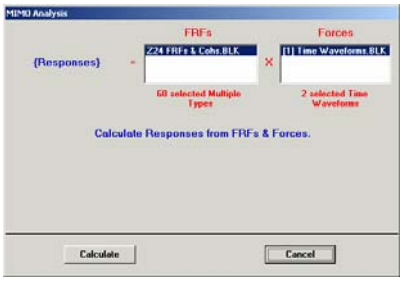
- Enter **[1] Time Waveforms** as the new file name and click on **OK**. The **[1] Time Waveforms.BLK** window will open.
- Close the **Z24 Bridge 2 Shaker Test Time Data.BLK** window.

MIMO Response Time Waveforms

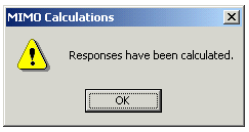
To calculate the response Time Waveforms of the bridge at **15** DOFs due to random forces **:1Z[1]** and **:2Z[1]**:

Select **M#1** and **M#2** in the **[1] Time Waveforms.BLK** Spreadsheet.

Execute: **Transform | MIMO | Responses** (from either open window). The **MIMO Analysis** dialog will open.



- Select **Z24 FRFs & Cohs.BLK** as the **FRFs** source.
- Select **[1] Time Waveforms.BLK** as the **Forces** source. Note that **2** (selected) Time Waveforms will be used.
- Press the **Calculate** button. When the Responses have been calculated, the **MIMO Calculations** dialog will open.



- Press **OK**. The **Data Block Selection** dialog will open.

- Press the **Add To** button, adding the **15** calculated responses to the **[1] Time Waveforms.BLK** Data Block.

Comparing Responses

To compare the calculated responses to the measured responses:

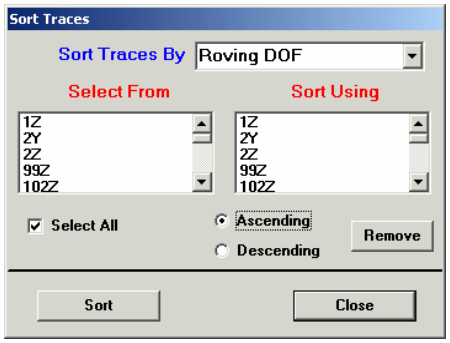
- Minimize the **Z24 FRFs & Cohs.BLK** window.

In the **[1] Time Waveforms.BLK** Spreadsheet:

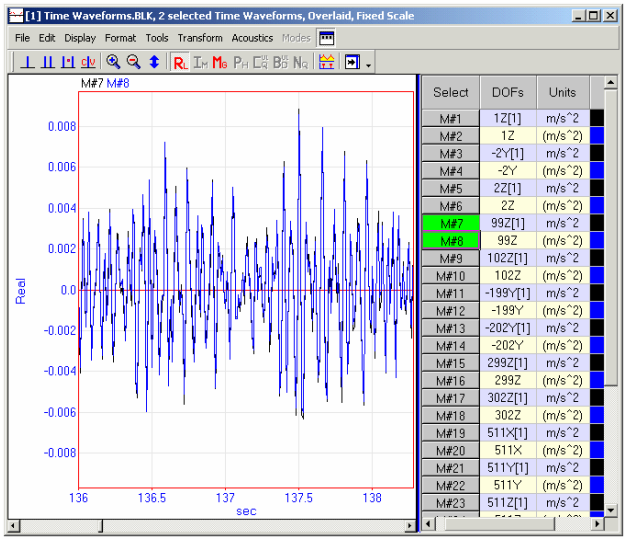
- Select the calculated Responses **M#18** through **M#32**
- Double click on the **Color** column header. The **Trace Color** dialog will open.

- Select **Single Color** and press **OK**. The **Color** dialog will open.
- Select **bright blue** and press **OK**.
- Double-click on the **Select** column Header to clear all selections.

Execute: **Edit | Sort Traces | By**. The **Sort Traces** dialog will open.



- Select **Roving DOF** from the **Sort Traces By** list.
- *Check* **Select All** and *click* on **Ascending**.
- Press the **Sort** button and then press the **Close** button.



*Comparing **calculated** and **measured** Responses.*

- Compare calculated and measured Time Waveforms for like DOFs by selecting adjacent **black** and **blue** Traces as shown above.

Execute: **Format | Overlay Traces**.

To expand the time axis to view the Traces in greater detail:

Execute: **Display | Zoom**.

- Move the cursor into the plot area where it will change to a *Zoom cursor* (⇔).
- Move the Zoom cursor to the desired left-side of the display, hold down the left mouse button and *drag* the Zoom cursor to the desired right-side of the display.
- Release the left mouse button. The display will Zoom between the limits set by the Zoom cursor.

Note that the calculated Response (blue) closely matches the actual Response (black) measured in the test.

To restore the full span of the display:

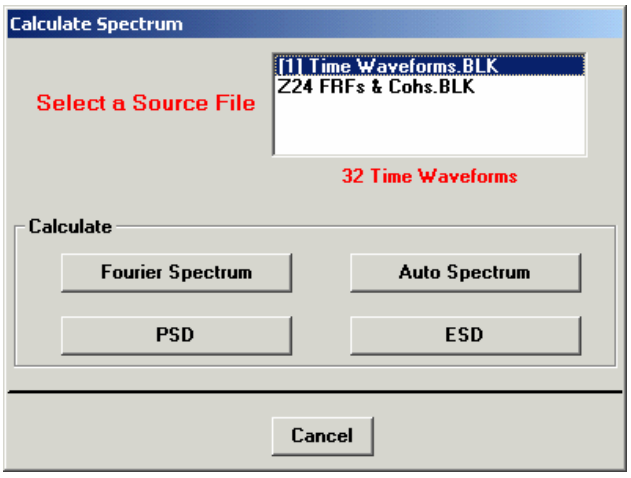
Hold down the *shift* key and execute: **Display | Mooz**.

PSD Comparison

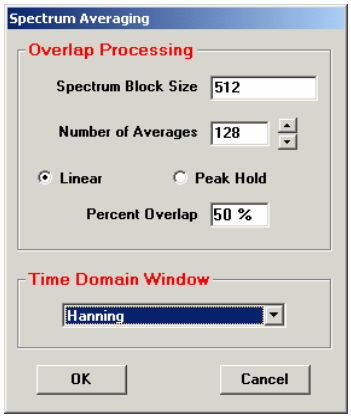
To calculate the Responses in the frequency domain:

- Unselect all Traces.

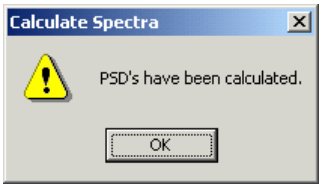
Execute: **Transform | Spectrum**. The **Calculate Spectrum** dialog will open.



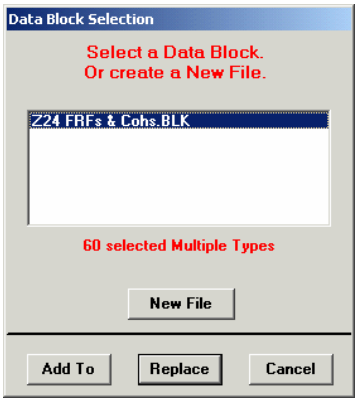
- Select **[1] Time Waveforms.BLK** as the Source File.
- Press the **PSD** button. The **Spectrum Averaging** dialog will open.



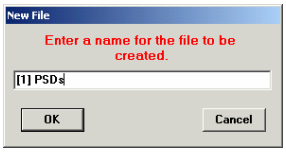
- Enter **512** as the **Spectrum Block Size**.
- Enter **128** as the **Number of Averages**.
- Select **Linear** averaging.
- Select the **Hanning** window.
- Press **OK**. The **Calculate Spectra** dialog will open.



- Press **OK**. The **Data Block Selection** dialog will open.



- Press the **New File** button. The **New File** dialog will open.



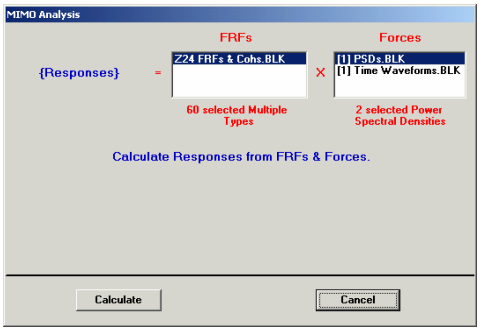
- Enter **[1] PSDs** as the new file name click on **OK**. The **[1] PSDs** window will open.
- Minimize the **[1] Time Waveforms.BLK** window.

The **[1] PSDs** Data Block contains the PSDs of the forces **:1Z[1]** and **:2Z[1]** and PSDs calculated from the **calculated** and **measured** Response time waveforms.

Next, we will use the **MIMO | Response** Command again to calculate the *Response PSDs directly from the Force PSDs*.

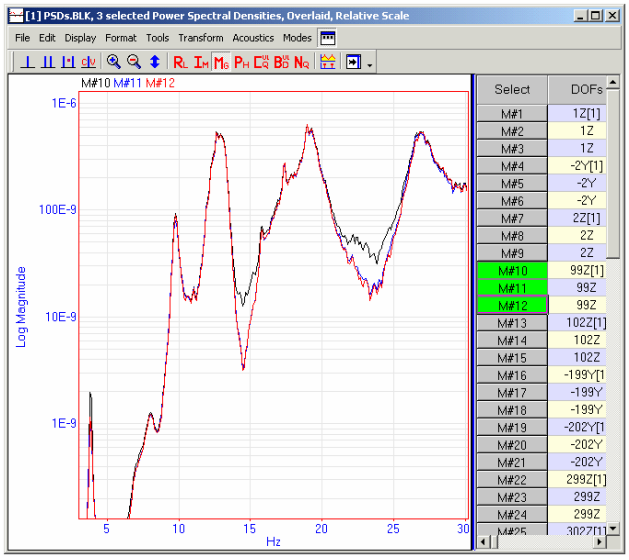
- Select **M#31** (**:2Z[1]**) and **M#32** (**:Z1[1]**).

Execute: **Transform | MIMO | Responses**. The **MIMO Analysis** dialog will open.



- Select **[1] PSDs.BLK** as the **Forces** source and press the **Calculate** button. the **MIMO Calculations** dialog will open.
- Press the **OK** button. The **Data Block Selection** dialog will open.
- Select **[1] PSDs** as the destination Data Block and press the **Add To** button. The 15 Response PSDs calculated directly from the Force PSDs are now added to the **[1] PSDs** Data Block.
- Select the added Response PSDs, **M#33** through **M#47**, double-click on the **Color** column Header and select **red** for these Traces.

Execute: **Edit | Sort Traces | By** and sort the Traces by **Roving DOFs**.



Comparison of measured and calculated Response PSDs.

Execute: **Format | Horizontal Axis**. Set the **Starting Value** to **3** and the **Span** to **27** to match the excitation frequency band used in the test.

- Compare the results by selecting adjacent **black**, **blue** and **red** Traces with the same Response DOF.

Note that the **red** Traces (Response PSDs calculated from Force PSDs) are virtually identical to the **blue** Traces (PSDs of calculated Response time waveforms). More importantly, note that both of these Calculated Responses closely match the PSDs of the measured responses (**black** Traces) from the test.

USING A SHAPE TABLE TO SYNTHESIZE FRFs

So far, we have used a *Data Block* containing the FRF elements of the MIMO model for calculating Responses. The **MIMO | Response** command also allows you to use a *Shape Table* to synthesize the required FRFs of the MIMO model. The Shape Table can contain either *Unit Modal Mass* (UMM) or *Residue* mode shapes.

Using UMM Mode Shapes

First, we will perform the response calculations using a Shape Table with UMM mode shapes in it.

- Close the **Z24 FRFs & Cohs.BLK** window.
- Minimize the **[1] PSDs.BLK** window.
- Open the **Z24 Bridge 8-mode fit.SHP** Shape Table.

Z24 Bridge 8-mode fit.SHP contains 8 mode shapes obtained by curve-fitting the Z2 bridge FRFs. These mode shapes are in UMM format. UMM mode shapes contain *displacement* response units. However, we can use The **MIMO | Response** command to calculate Responses *with acceleration, velocity or displacement units*.

Note that the mode shapes in the **Z24 Bridge 8-mode fit.SHP** file contain all **75** DOFs measured in the test. We only want to compare the **15** Response DOFs of Measurement Set [1].

Selecting DOFs in the Shape Table

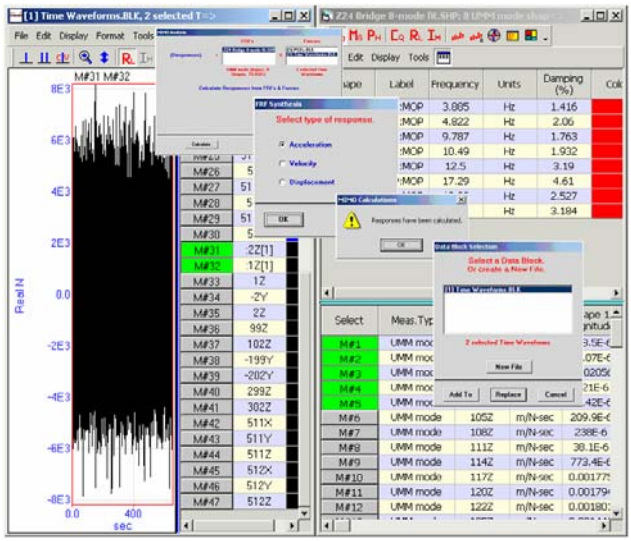
- Open the **[1] Time Waveforms.BLK** window.

In the **Z24 Bridge 8-mode fit.SHP** Spreadsheet:

- Select the **15** DOFs that match the Response DOFs in the **[1] Time Waveforms.BLK** Spreadsheet.

Calculating Response Time Waveforms

- Select **M#31** and **M#32** (the :2Z[1] and :1Z[1] Forces) in the **[1] Time Waveforms.BLK** window.

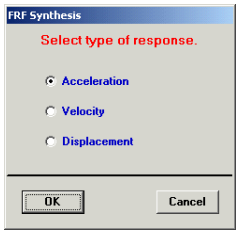


Calculating Responses using a UMM Shape Table.

Execute: **Transform | MIMO | Responses**. The **MIMO Analysis** dialog will open.

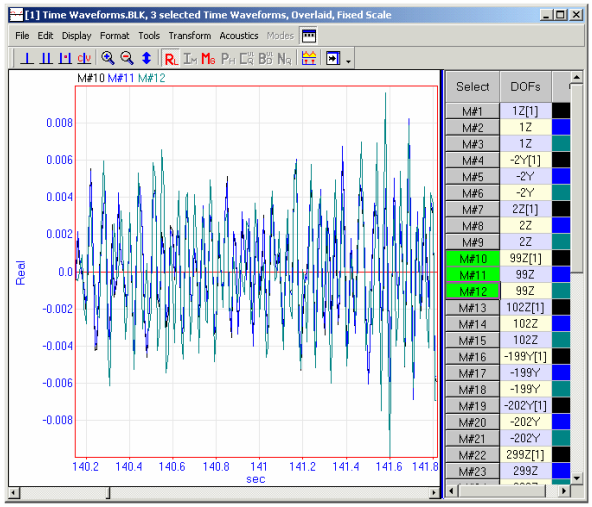
- Select the **Z24 Bridge 8-mode fit.SHP** Shape Table as the source of **FRFs**.
- Select **[1] Time Waveforms** as the source of **Forces**.

- Press the **Calculate** button. The **FRF Synthesis** dialog will open.



- Select **Acceleration** as the desired type of response and click on **OK**. The **MIMO Calculations** dialog will open when the calculation is done.
- Click on **OK**. The **Data Block Selection** dialog will open.
- Select the **[1] Time Waveforms.BLK** Data Block and press the **Add To** button. The **15** new Response Time Waveforms are added to the end of the Data Block.
- Minimize the **Z24 Bridge 8-mode fit.SHP** window.
- Select the added Response Traces, **M#33** through **M#47**, double-click on the **Color** column Header and select **green** for these Traces.

Execute: **Edit | Sort Traces | By** and sort the Traces by **Roving DOFs**.



***Shape-based**, **FRF-based** and **measured** Responses overlaid.*

Execute: **Format | Overlay Traces** and Zoom the display as you did with prior overlays.

- Compare the results by selecting adjacent **black** (measured), **blue** (FRF-based) and **green** (Shape-based) Response Traces with the same Response DOF.

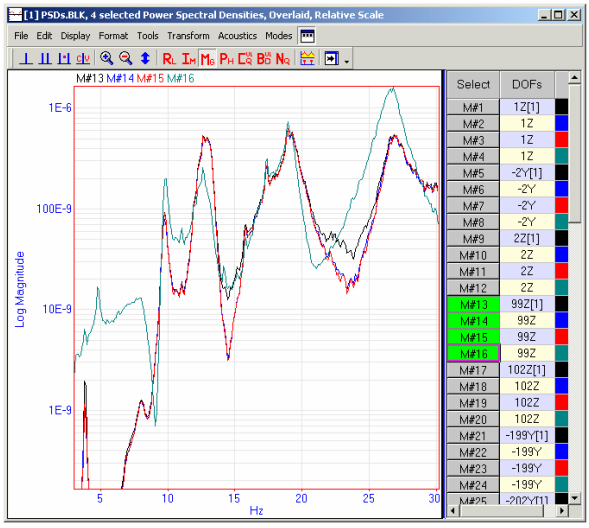
Calculating Response PSDs

- Minimize the **[1] Time Waveforms.BLK** window.
- Open the **[1] PSDs.BLK** window.
- Select **M#46** (**:2Z[1]**)and **M#47** (**:1Z[1]**) measured force signals.

Execute: **Transform | MIMO | Responses**. The **MIMO Analysis** dialog will open.

- Select **[1] PSDs.BLK** as the source of **Forces** and press the **Calculate** button.
- Follow the remaining steps just discussed in **Calculating Response Time Waveforms** to add **15** new Response PSDs to the **[1] PSDs.BLK** as **M#48** through **M#62**.
- Change the Color of **M#48** through **M#62** to **green** and **Sort** the Traces by **Roving DOFs**.

Overlay the results as shown below.



UMM PSD results overlaid with prior calculations.

Notice that the **Shape-based** Responses that similar to the **measured** Responses, but don't match the measured Responses as well as the **FRF-based** Responses.

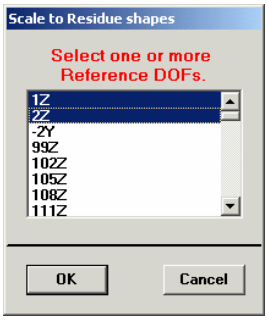
Using Residue Mode Shapes

Residue mode shapes are normally created by saving shapes during curve fitting, but they can also be created by rescaling UMM mode shapes.

We will now scale the UMM mode shapes in **Z24 Bridge 8-mode fit.SHP** to Residue mode shapes and repeat the Response calculations.

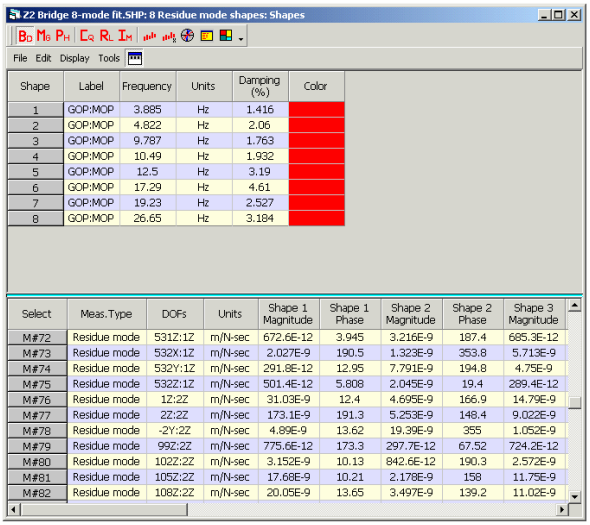
In the **Z24 Bridge 8-mode fit.SHP** window:

Execute: **Tools | Scaling | Residues**. **The Scale to Residue shapes** dialog will open.



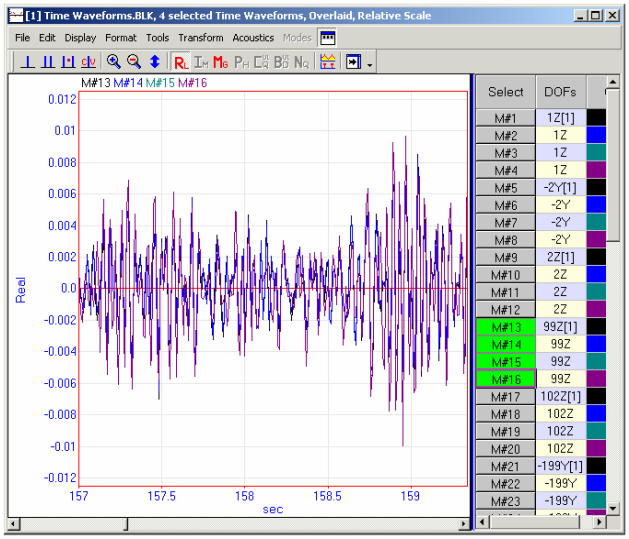
Select **:1Z** and **:2Z** as the Reference DOFs (where the Forces are applied):

- *Click* on **1Z**.
- Hold down the *Control* key and *click* on **2Z**.
- *Click* on the **OK** button. The Shape Table will now contain Residue mode shapes. Each mode shape has 150 DOFs, 75 for each Reference DOF.

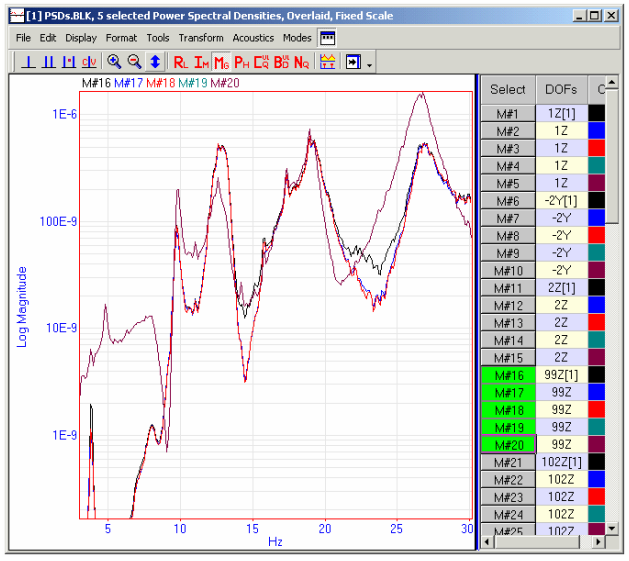


Residue Mode Shapes with **:1Z** *&* **:2Z** *References.*

- Repeat the previous steps used with the UMM Mode Shapes, starting by selecting the same DOFs from the modal model.
- Save the resulting work as **purple** Traces **M#48** through **M#62** in the **[1] Time Waveforms.BLK** file and as **purple** Traces **M#63** through **M#80** in **[1] PSDs.BLK** file.
- Compare your work with the following two figures.



Overlay of all Time Waveforms for one Response DOF.



Overlay of all PSDs for one Response DOF.

SUMMARY

You have carried out MIMO Response calculations defined by equation (1) in several ways. You have:

1. Used a Data Block of FRFs to define the FRF matrix.
2. Used UMM mode shapes to synthesize FRF matrix elements.
3. Used Residue mode shapes to synthesize FRF matrix elements.
4. Calculated responses to Force Time Waveforms
5. Calculated responses to Force PSDs.

All of these approaches yielded very comparable results.

CONCLUSIONS

The **FRF-based** calculated Responses matched the actual Measurement Set [**1**] Responses better than the **Shape-based** responses. This is because the FRFs contain the residual effects of all modes of the structure, not just the **8** modes used in the modal model.

The **UMM** and **Residue** mode shape results are *identical* to one another and are smoother curves because they don't contain the "noise" contributions of the FRF-based calculations.

It should also be noted that **FRF-based** calculations *interpolate* the FRFs to match the frequency-axis parameters of the excitation forces. On the other hand, when the Response is calculated using a Shape Table, the required FRFs are synthesized to match the frequency-axis of the forces.