



ME'scopeVES Application Note #22

Calculating Sinusoidal Responses of MIMO Systems

INTRODUCTION

In this note, MIMO Commands will be used to calculate responses of a structure due to *simultaneously applied sinusoidal forces*. These forces may be applied at *different locations*, acting in *different directions* with *different magnitudes* and *different phases*.

Driving forces and response motions of a vibrating structure are related in a very straightforward manner when the motions and forces are described in the Frequency Domain. The motions at N DOFs (points and directions) on the structure are related to forces applied to M DOFs by the following matrix of NM Frequency Response Functions (FRFs). Specifically:

$$\{X(f)\}_{Nx1} = [H(f)]_{NxM} \cdot \{F(f)\}_{Mx1} \quad (1)$$

ME'scopeVES contains commands for investigating all aspects of the Multiple-Input Multiple Output (MIMO) relationship of equation (1). You can:

1. **Calculate all the FRFs** in the matrix from measured Forces and Responses. See Application Note #20.
2. **Calculate multiple Responses**, given a known matrix of FRFs and a vector of Forces. See also Application Note #21.
3. **Calculate multiple Forces**, given a known matrix of FRFs and a vector of Response motions.

In other words, *provide any two elements of equation (1) and the third can be calculated using MIMO commands*.

The FRF matrix can either be provided from a *Data Block* of FRF measurements, or synthesized from a *Shape Table* containing the structure's mode shapes. Both of these cases are covered in this note.

In this note, we will focus on calculating the *Responses*, given a known FRF matrix and a Force vector that contains *sinusoidal forces* at a *single frequency*.

Steps in the application note can be duplicated using VT-550 Visual Modal Pro or any package that includes option VES-350 Advanced Signal Processing.

EXAMPLE DATA

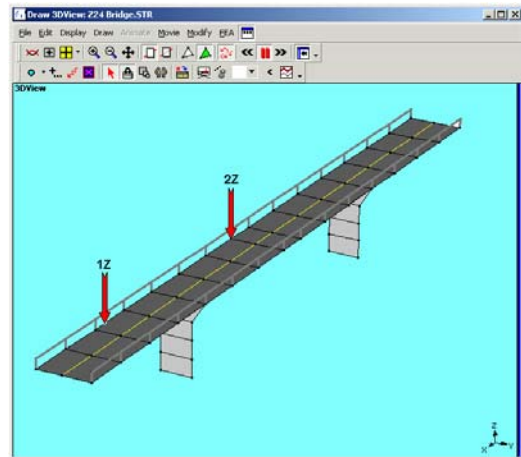
This Application Note requires the FRFs that are calculated in Application Note #20. They are also provided in the More Examples sub-directory on your Installation CD.

 Open ME'scopeVES.

 Execute: **File | Project | Open**

- Select **My Z24 Bridge.PRJ** from the **More Examples** subdirectory.

This will open the **Z24 FRFs & Cohs.BLK** Data Block containing *Frequency Response Functions* (FRFs), *Multiple Coherence Functions* and *Partial Coherence Functions* calculated for the Z24 bridge. The project also contains the Structure file, **Z24 Bridge.STR**, the Shape Table file, **Z24 Bridge 8-mode fit.SHP**, and the Data Block file, **Z24 Bridge 2 Shaker Test Time Data.BLK**, containing the measured time domain signals.



3D model showing shaker drive-site locations.

Z24 FRFs & Cohs.BLK contains **495** Traces in nine Measurement Sets. The Traces include:

198 FRFs - 144 *unique* FRFs and 6 *redundant* FRFs measured 9 times each (once per Measurement Set).

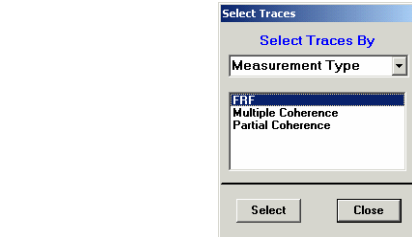
99 Multiple Coherences – one for each Response.

198 Partial Coherences– two for each Response.

PREPARING THE FRFs

First, select the FRFs in the **Z24 FRFs & Cohs.BLK** Data Block:

Execute: **Edit | Select Traces | By**. The **Select traces** dialog will open.



- Select **Measurement Type** from the **Select Traces By** list and click on **FRF** in the list below it.
- Press **Select**. (Note that **150** Traces are selected.)
- Press **Close** to exit the dialog.

The FRFs are now ready for MIMO forced Response calculation. To more clearly observe the resonance peaks in the FRFs:

Execute: **Display | Imaginary** to view the imaginary parts of the FRFs.

Execute: **Format | Horizontal Axis**. Set the **Starting Value** to **5 Hz** and the **Span** to **10 Hz**.

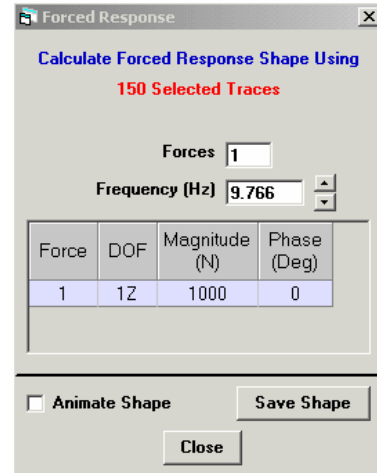
Execute: **Format | Overlay Traces**.

Execute: **Display | Cursor | Line Cursor**.

Execute: **Display | Cursor | Cursor Values**.

CALCULATING SINUSOIDAL RESPONSES

Execute: **Transform | MIMO | Sinusoidal Response**. The **Forced Response** dialog will open.



In the **Forced Response** dialog:

1. Enter **1** as the number of **Forces**.
2. Adjust the **Frequency (Hz)** to be **9.766** by clicking the up/down arrows *or moving the Line Cursor* in the **Z24 FRFs & Cohs.BLK** window.
3. Select the **DOF** to be **1Z**.
4. Enter **1000** as the force **Magnitude (N)**.
5. Enter **0** as the **Phase (Deg)**.

The Forced Response may now be used to *animate* the Structure or it can be *saved* as a Shape Table.

ANIMATING THE RESPONSE SHAPE

- Open the **Z24 Bridge.STR** file.

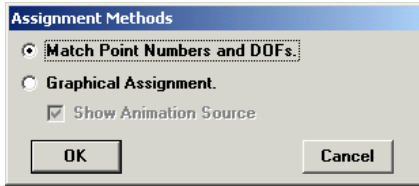
Execute: **Window | Arrange | For Animation**.

- *Check* the **Animate Shape** box.

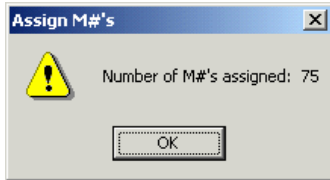
Before animating the Forced Response Shape *for the first time*, you must assign Measurement Numbers (**M#s**) to the DOFs of the Structure and set up the display Interpolation Equations.

To assign **M#s** to the Structure's DOFs:

Execute: **Assign | M#'s**. The **Assignment Methods** dialog will open.



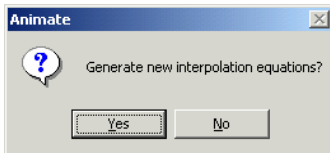
- Choose **Match Point Numbers and DOFs** and click **OK**. When the M#'s have been assigned, the following dialog box will open.



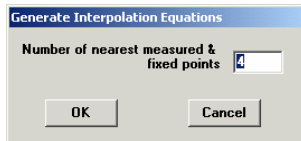
- Click on **OK**.

Next, setup the Interpolation Equations.

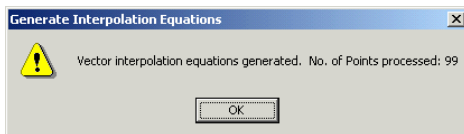
- Press the **Animate Shape** button in the **Forced Response** dialog. The **Animate** dialog will open.



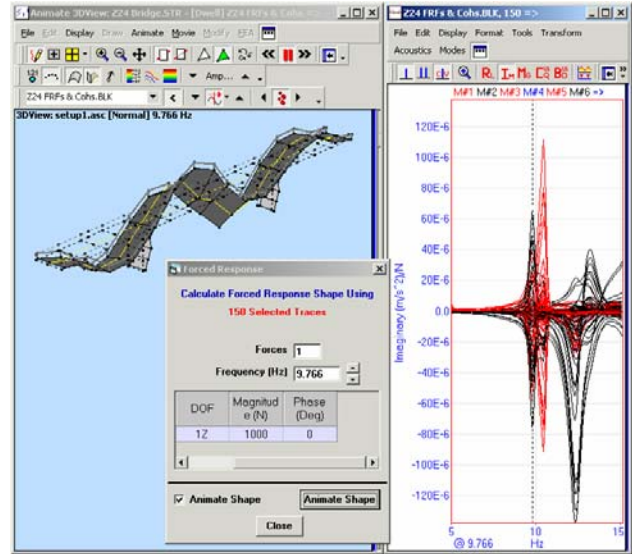
- Click on **Yes** to generate new Interpolation Equations. The **Generate Interpolation Equations** dialog will open.



- Enter **4** and press **OK**. A **Generate Interpolation Equations** summary message will open.



- Press **OK**. The FRF animation will start.



Bridge response to 9.766 Hz 1000 N force applied at :1Z.

- To stop the animation, *un-check* the **Animate Shape** box in the **Forced Response** dialog.

EXPLORING THE BRIDGE RESPONSES

We will now explore the bridge's Response to Forces at different frequencies and different locations.

Note: You can make changes on the **Forced Response** dialog box at any time during an animation. However, these changes do not become effective until you press the **Animate Shape** button.

Applying a sinusoidal force at **9.766 Hz** to DOF **:1Z** results in a complicated response pattern (above) that is dominated by bending and torsional modes as well as some lateral sway.

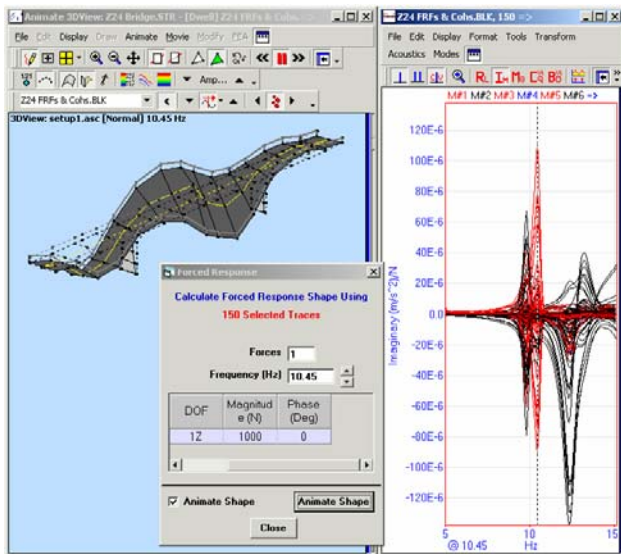
The overlaid FRFs in the Data Block window explain this. The peaks at **9.766 Hz** correspond to a bending mode of the deck with nodes at the ends, the center and the 1/4 span support piers. The black FRF Traces (reference DOF **:1Z**) have the largest peaks at 9.766 Hz, showing strong response to **:1Z** excitation at this frequency.

Another resonance can be seen at **10.45 Hz**. This is a torsional mode of the bridge. The red FRF Traces (reference DOF **:2Z**) have the largest peaks in this mode. This indicates the torsion mode is very responsive to **:2Z** excitation at 10.45 Hz.

However, the bridge response can be dominated by the torsional mode when a force is applied at **:1Z** as well. To observe this:

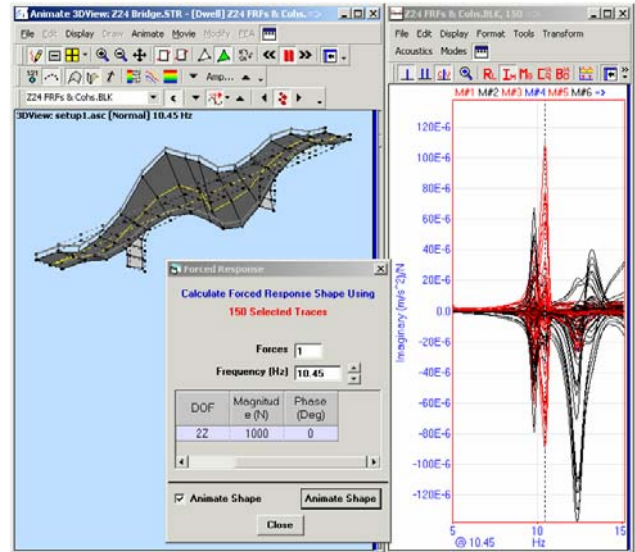
- Increase the **Frequency (Hz)** setting in the Forced Response panel to **10.45 Hz**.
- Press the **Animate Shape** button.

Note the change in the Response Shape. Twisting of the center span is now much more evident and the contribution of the bending mode is significantly reduced.



10.45 Hz sinusoidal excitation at DOF **:1Z**.

The 9.766 Hz bending and 10.45 Hz torsional modes exhibit *closely coupled* behavior. While these are *distinct modes* of the bridge, exciting at the natural frequency of either one will produce motion that is a combination of both mode shapes. The combination changes with the DOF that is excited. For example, the following figure illustrates the response to a 1000 N sinusoidal force of 10.45 Hz applied at DOF **:2Z**.



10.45 Hz sinusoidal excitation applied at DOF **:2Z**.

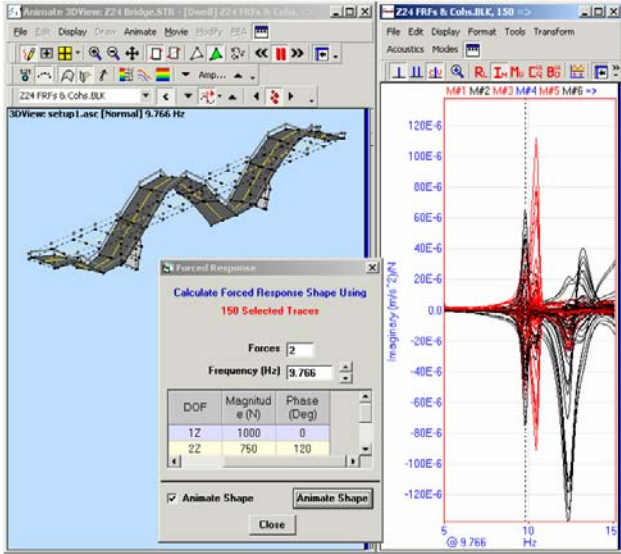
Applying Multiple Forces

Next, let's apply two 9.766 Hz forces at DOFs **:1Z** and **:2Z** *simultaneously*. We will use forces of different *magnitudes* and *phases*.

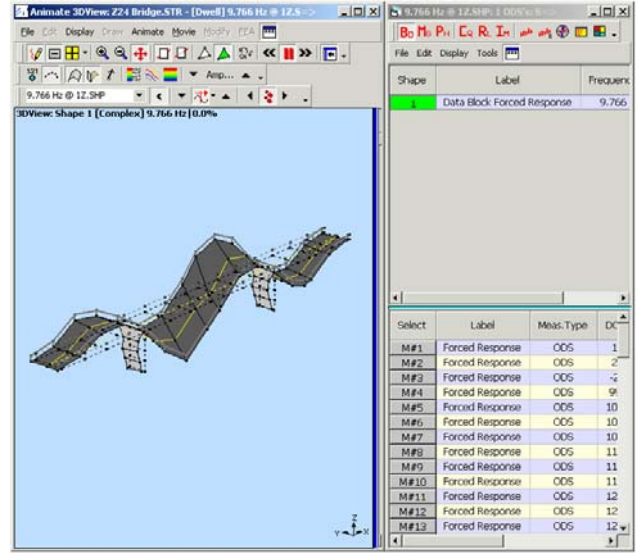
In the **Forced Response** dialog box:

- Enter **2** as the number of **Forces**.
- Set the Frequency (Hz) to 9.766.
- In the spreadsheet, apply Force **1** to DOF **1Z** with a Magnitude (N) of **1000** and a Phase (Deg) of **0**.
- Apply Force **2** to DOF **2Z** with a Magnitude (N) of **750** and a Phase (Deg) of **120**.
- Press the **Animate Shape** button.

Because the magnitudes and phases of the two forces correspond more closely with the 9.766 Hz bending mode of the bridge, it dominates the forced response. There is little or no contribution from the 10.45 Hz torsion mode.



Applying two forces at 9.766 Hz.



Animating from the 9.766 Hz tuned dwell.SHP file.

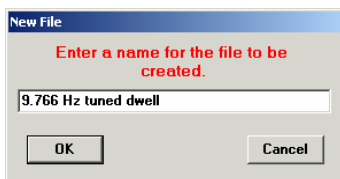
SAVING A FORCED RESPONSE SHAPE

To save this Forced Response as a Shape file:

- Un-check the **Animate Shape** box in the **Forced Response** dialog. The animation will stop and the **Save Shape** button will appear.
- Press the **Save Shape** button. The **Shape Table Selection** dialog will open.



- Press the **New File** button. The **New File** dialog will open.



- Enter **9.766 Hz tuned dwell** as the new file name and click on **OK**. The **9.766 Hz tuned dwell.SHP** window will open.

- Close the **Forced Response** dialog.
- Close the **Z24 FRFs & Cohs.BLK** window.

To animate the Structure using the **9.766 Hz tuned dwell.SHP** Shape Table:

- Execute: **Assign | M#'s. Respond** with **OK** to the two dialogs which open.
- Execute: **Animate the Structure**. Respond with **Yes, OK** and **OK** to the three **Interpolation Equations** dialogs which open. The animation will begin.

To stop the animation:

- Execute: **Draw the Structure**.

Close the **9.766 Hz tuned dwell.SHP** window.

USING SHAPE FILES TO SYNTHESIZE FRFs

The FRF matrix of equation (1) can be also be synthesized using a modal model contained in a *Shape Table*. Sinusoidal forced responses can be calculated using Shape Tables that contain either *Unit Modal Mass* (UMM) or *Residue* mode shapes. The FRF matrix is automatically derived by synthesizing the required FRF elements from the mode shapes.

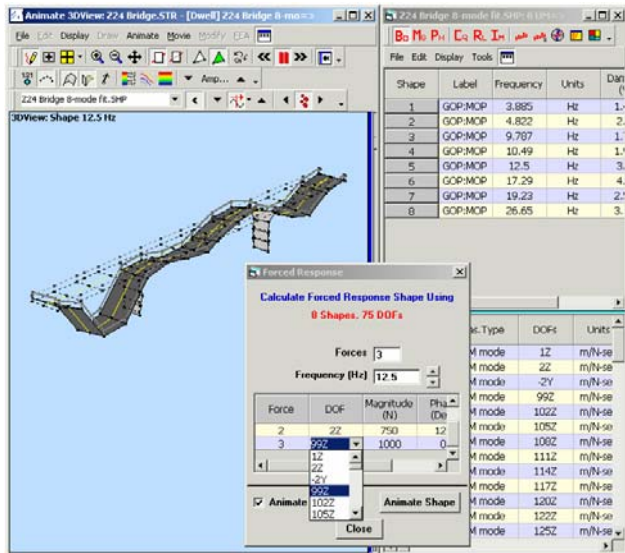
Using UMM Mode Shapes

To animate Forced Responses using UMM mode shapes of the Z24 bridge:

- Open **Z24 Bridge 8-mode fit.SHP**.

In the **Z24 Bridge 8-mode fit.SHP**:

 Execute: **Tools | Sinusoidal Response**.




Animating forced response using a UMM Shape File.

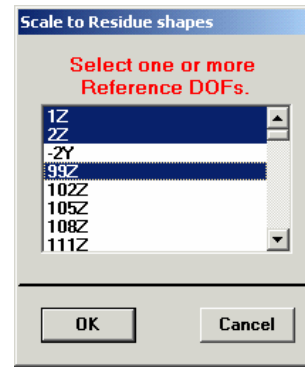
Sinusoidal response shapes are animated in exactly the same way as previously described in the **CALCULATING SINUSOIDAL REPOSES** section. When UMM mode shapes are used, notice that forces can be applied at any DOF of the shapes.

Using Residue Shapes


Residue mode shapes are normally created by saving shapes during curve fitting, but they can also be created by rescaling UMM mode shapes.

- Close the **Forced Response** dialog.

 Execute: **Tools | Scaling | Residues**. The **Scale to Residue Shapes** dialog will open.



- Hold the *Control* key down and *click* on all desired **Reference DOFs** to which you may wish to apply forces. (**1Z**, **2Z** and **99Z**, for example.)
- Click on **OK**. The Shape Table now contains *Residue* mode shapes.

 Execute: **Tools | Sinusoidal Response**. The Forced Response panel will open.

SUMMARY

Responses to single and multiple sinusoidal forces were calculated using the **Transform | MIMO | Sinusoidal Response** command in a Data Block of FRFs. Responses were also calculated using the **Tools | Sinusoidal Response** command with both UMM and Residue Mode Shapes from a Shape Table. The Forced Responses were animated and also saved into a new Shape Table.