



## Calculating Force Inputs of MIMO Systems

### INTRODUCTION

In this note, a MIMO Command will be used to calculate multiple *Forces*, given a known FRF matrix and a Response vector.

Driving forces and response motions of a vibrating structure are related in a very straightforward manner when the motions and forces are described in the Frequency Domain. The motions at  $N$  DOFs (points and directions) on the structure are related to forces applied to  $M$  DOFs by the following matrix of  $NM$  Frequency Response Functions (FRFs). Specifically:

$$\{X(f)\}_{Nx1} = [H(f)]_{NxM} \cdot \{F(f)\}_{Mx1} \quad (1)$$

**ME'scopeVES** contains commands for investigating all aspects of the Multiple-Input Multiple Output (MIMO) relationship of equation (1). You can:

1. **Calculate all the FRFs** in the matrix from measured Forces and Responses. See [Application Note #20](#).
2. **Calculate multiple Responses**, given a known matrix of FRFs and a vector of Forces. See also [Application Note #22](#).
3. **Calculate multiple Forces**, given a known matrix of FRFs and a vector of Response motions.

In other words, *provide any two elements of equation (1) and the third can be calculated using MIMO commands.*

The FRF matrix can either be provided from a *Data Block* of FRF measurements, or synthesized from a *Shape Table* containing the structure's mode shapes. Both of these cases are covered in this note. Forces will be calculated as both Time Waveforms and PSDs, and will be compared with measured results.

**NOTE:** Steps in the application note can be duplicated using **VT-550 Visual Modal Pro** or any package that includes option **VES-350 Advanced Signal Processing**.

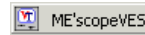
### USING A DATA BLOCK OF FRFs

We will use test data from the Z24 bridge in our example. As described in [Application Note #20](#), this MIMO data was acquired in **9 Measurement Sets** using two shakers with random noise excitation.



Z24 Bridge crosses Bern-to-Zurich Swiss highway A1.

This Application Note requires the FRFs that are calculated in [Application Note #20](#). They are also provided in the [More Examples](#) sub-directory on your Installation CD.

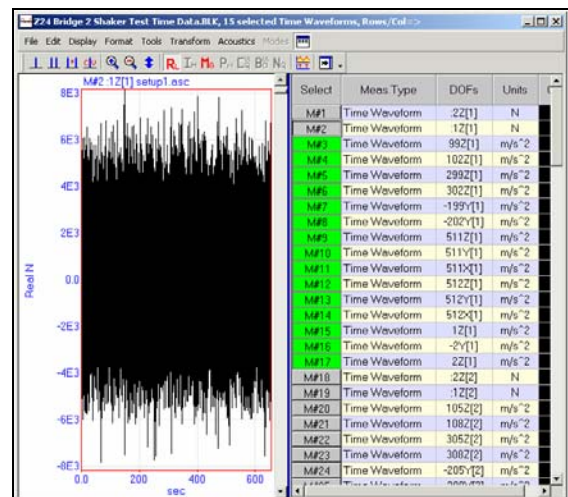


Open **ME'scopeVES**.



Execute: **File | Project | Open**

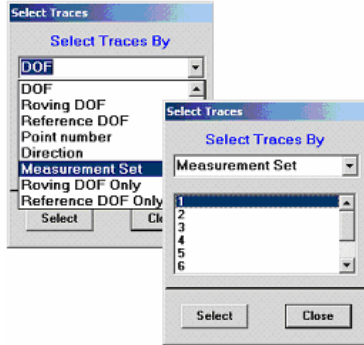
- Select **My Z24 Bridge.PRJ** from the **More Examples** subdirectory.
- Open the **Z24 Bridge 2 Shaker Test Time Data.Blk** Data Block, shown below.



Measurement Set [1] Responses Selected in Data Block.

## COPYING FORCE & RESPONSE TRACES

Execute: **Edit | Select Traces | By**. The **Select Traces** dialog will open.

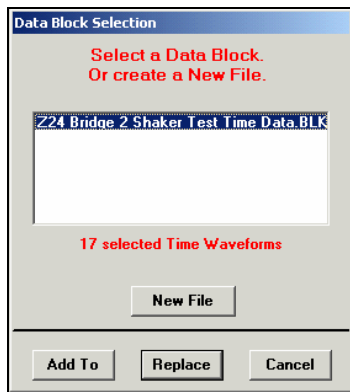


- Select **Measurement Set** and **1** in the in the drop-down menus and press the **Select** button. **17** Traces will be selected.
- Press **Close** to close the **Select Traces** dialog.

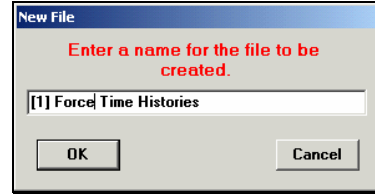
Of the **17** Traces just selected, two (**M#1** and **M#2**) are the random forces (**:2Z[1]** and **:1Z[1]**) that were applied to the bridge during Measurement Set **[1]**. The remaining **15** Traces are the resulting acceleration Responses that were simultaneously measured.

*These 17 traces are a consistent set of 2-input, 15-output MIMO measurements. We will use the 15 Responses to calculate the 2 Forces. To copy the selected Traces to a new file:*

Execute: **Copy | Traces**. The **Data Block Selection** dialog will open.



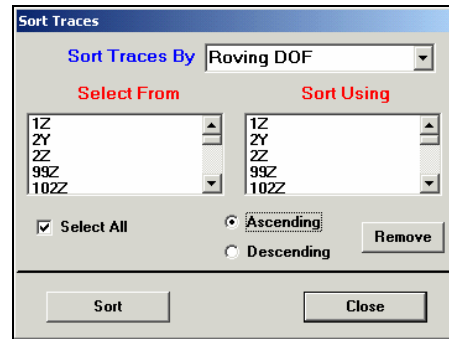
- Press the **New File** button. The **New File** dialog will open.



- Enter **[1] Force Time Histories** as the new file name and click on **OK**. The **[1] Force Time Histories.BLK** window will open.
- Close the **Z24 Bridge 2 Shaker Test Time Data.BLK** window.

To sort the Traces so that all of the Responses are at the top of the **[1] Force Time Histories** Traces Spreadsheet:

Execute: **Edit | Sort Traces | By**. The **Sort Traces** dialog will open.



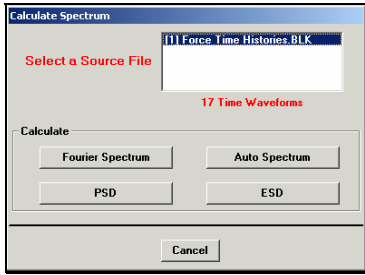
- Select **Roving DOF** from the **Sort Traces By** list.
- Check **Select All** and click on **Ascending**.
- Press the **Sort** button and then press the **Close** button.

## CALCULATING PSDs

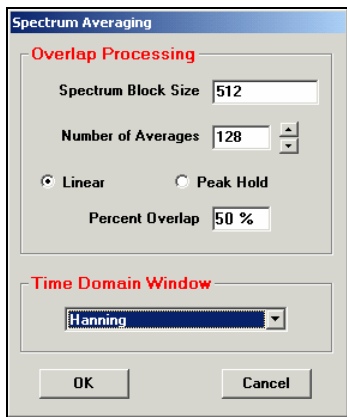
Both time domain Traces and Power Spectral Densities (PSD's) will be used for comparisons later on.

To calculate the PSDs:

Execute: **Transform | Spectrum**. The **Calculate Spectrum** dialog will open.



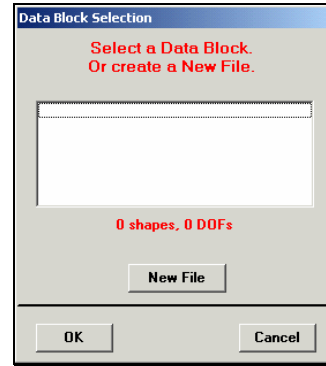
- Select **[1] Force Time Histories.BLK** as the Source File.
- Press the **PSD** button. The **Spectrum Averaging** dialog will open.



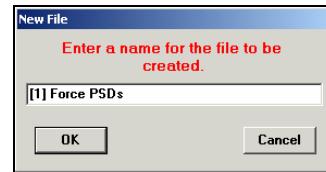
- Enter **512** as the **Spectrum Block Size**.
- Enter **128** as the **Number of Averages**.
- Select **Linear** averaging.
- Select the **Hanning** window.
- Press **OK**. The **Calculate Spectra** dialog will open.



- Press **OK**. The **Data Block Selection** dialog will open.



- Press the **New File** button. The **New File** dialog will open.



- Enter **[1] Force PSDs** as the new file name click on **OK**. The **[1] Force PSDs** window will open.
- Minimize the **[1] Force PSDs** window. We will use it shortly.
- Select Response measurements **M#1** through **M#15** in the **[1] Force Time Histories.BLK** Spreadsheet

**RETRIEVING THE FRFs**

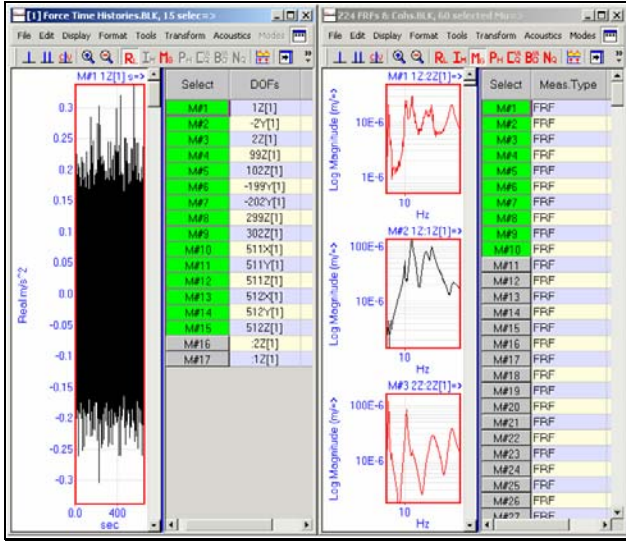
We now need to retrieve the **30** FRFs that relate the **15** Responses to the **2** Forces from the **Z24** bridge Data Block file with the FRFs in it.

- Open the **Z24 FRFs & Cohs.BLK** Data Block.

Execute: **Edit | Select Traces | By** and select **Measurement Set** and **1**.

Note (in the **Z24 FRFs & Cohs.BLK** Title Bar) that **60** Traces have been selected. Of these, **30** are the desired FRFs and the other **30** are the associated *Partial Coherences*.

**NOTE:** Only the FRFs are used in MIMO calculations. The Coherences are ignored.

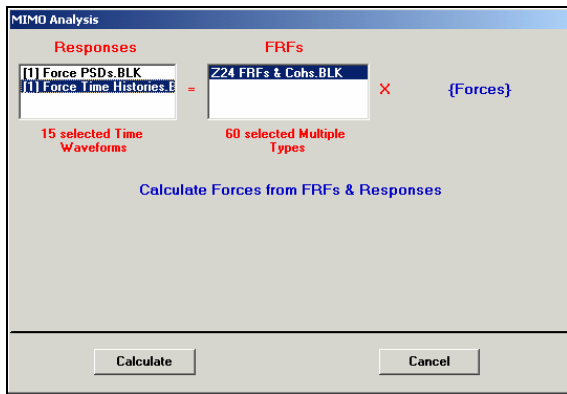


Responses and FRFs ready for MIMO Force calculation.

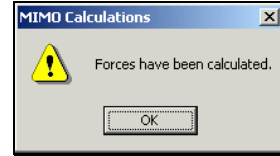
With the desired Responses and FRFs selected in two different Data Block windows, we are ready to calculate the Forces.

**CALCULATING FORCE TIME WAVEFORMS**

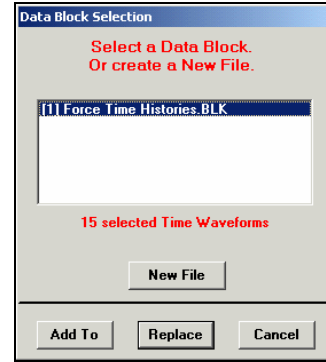
Execute: **Transform | MIMO | Forces** from either window. The **MIMO Analysis** dialog will open.



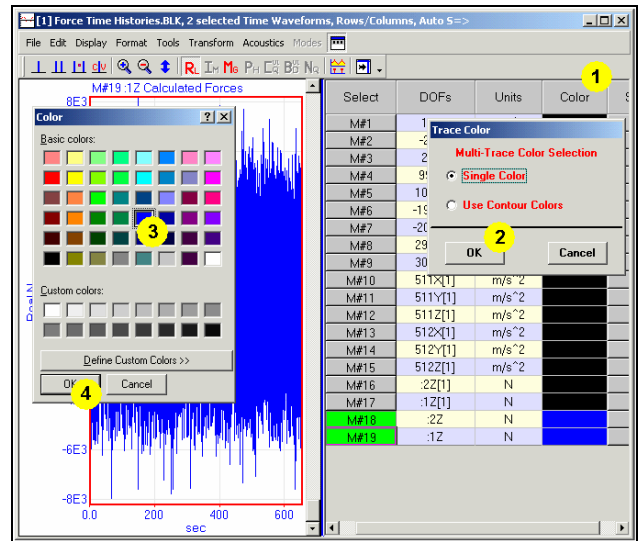
- Select **[1] Force Time Histories.BLK** as the **Responses** source. Note that **15** Time Waveforms will be used.
- Select **Z24 FRFs & Cohs.BLK** as the **FRFs** source. Note that **60** Traces of *multiple types* have been selected. (Only the **30** FRFs will be used.)
- Press the **Calculate** button. When the Force calculation is completed, the **MIMO Calculations** dialog will open.



- Press **OK**. The **Data Block Selection** dialog will open.



- Press the **Add To** button. The calculated forces are added to the bottom of the **[1] Force Time Histories.BLK** Spreadsheet as **M#18 (:2Z)** and **M#19 (:1Z)**.
- Minimize the **Z24 FRFs & Cohs.BLK** window.



Setting Trace Color of calculated Forces.

- Select **M#18** and **M#19**. Double-click on the **Color** column Header to change their Trace Color to **blue** as shown above.

To compare the calculated forces to the actual time domain measured forces:

- Select **M#17 (:1Z[1])** and **M#19 (:1Z)** Force Traces.

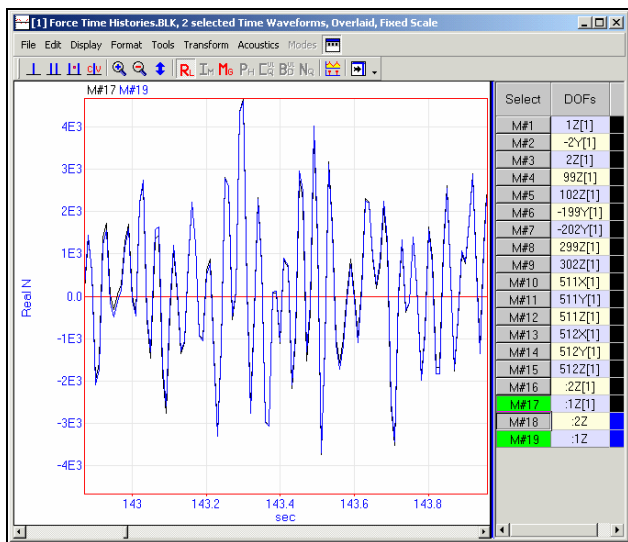


Execute: **Format | Overlay Traces.**



Execute: **Display | Zoom.**

- Move the cursor into the plot area where it will change to a *Zoom cursor* ( $\leftrightarrow$ ).
- Move the Zoom cursor to the desired left-side of the display, hold down the left mouse button and *drag* the Zoom cursor to the desired right-side of the display.
- Release the left mouse button. The display will Zoom between the limits of the Zoom cursor.



Expanded overlay of *calculated* and *measured* 1Z forces.

- Select **M#16** and **M#18** to compare the calculated Force **:1Z** with the measured Force **:1Z[1]**.
- Use the horizontal Scroll Bar to pan through the overlaid force Time Waveforms. The **calculated** Forces should closely match with the **actual** Forces (measured during the test).

To restore the full span of the display:



Hold down the *shift* key and execute: **Display | Mooz.**

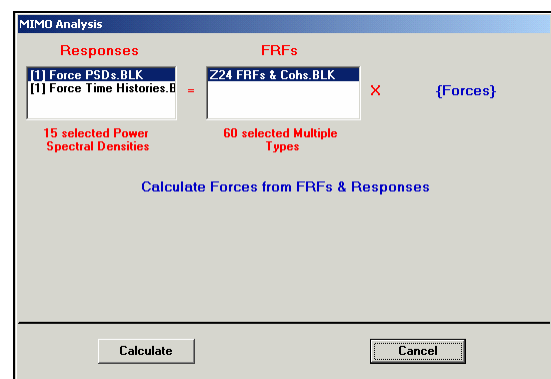
## CALCULATING FORCE PSDs

To calculate and compare Forces in the frequency domain:

- Minimize the **[1] Force Time Histories.BLK** window.
- Open the **[1] Force PSDs.BLK** window.
- Select the Response PSDs **M#1** through **M#15**.



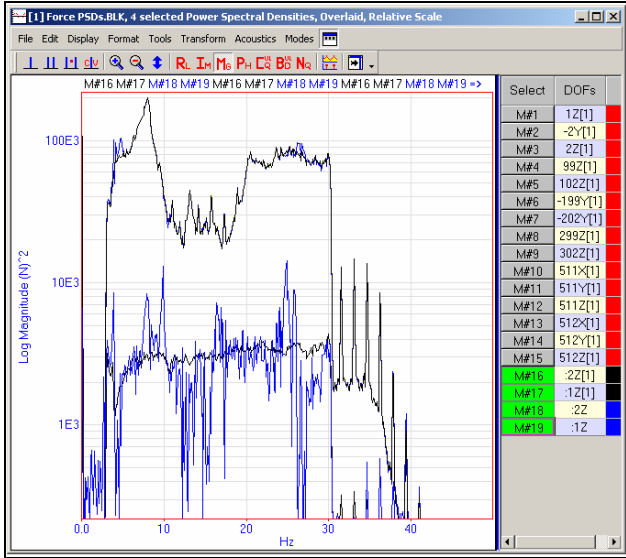
Execute: **Transform | MIMO | Forces.** The **MIMO Analysis** dialog will open.



- Select **[1] Force PSDs.BLK** as the source of **Responses** and press the **Calculate** button. The **MIMO Calculations** dialog will open.
- Press **OK**. The **Data Block Selection** dialog will open.
- Press the **Add To** button. The calculated forces are added to the bottom of the **[1] Force PSDs.BLK** Traces Spreadsheet as **M#18 (:2Z)** and **M#19 (:1Z)**.
- Select **M#18** and **M#19** and change their Trace Color to **blue**.
- Select the Force Traces **M#16 (:2Z[1])**, **M#17 (:1Z[1])**, **M#18 (:1Z)** and **M#19 (:1Z)**.



Execute: **Format | Overlay Traces.**

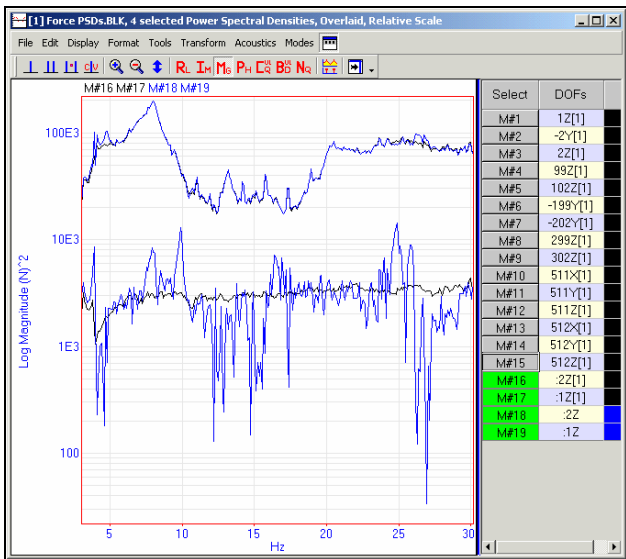


0-50 Hz overlay of *calculated* and *measured* force PSDs

During the test, the shakers were driven by random noise, with a “flat” spectrum from 3 Hz to 30 Hz. The shakers were run “open loop” (without force control feedback). The mechanical impedance mismatch between the shakers and the bridge results in Force spectra that are not perfectly “flat”, but are clearly dominated by power in the 3 to 30 Hz band.



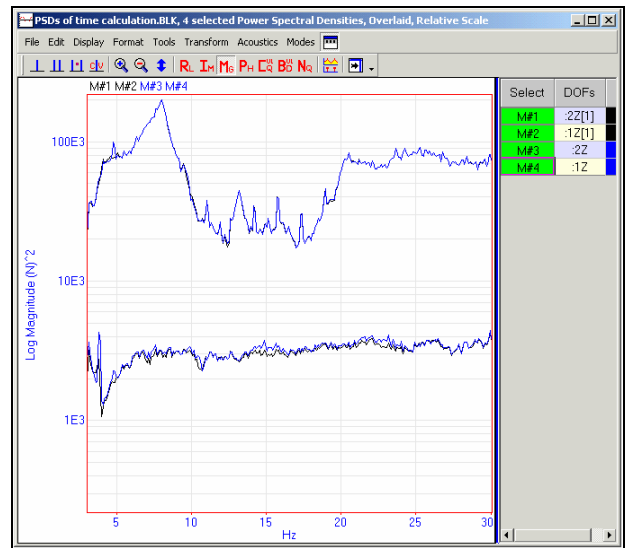
Execute: **Format | Horizontal Axis**. Set the **Starting Value** to 3 Hz and the **Span** to 27 Hz to match the excitation bandwidth used in the test.



3-30 Hz overlay of *calculated* and *measured* force PSDs.

Note the excellent match between the :1Z and :1Z[1] PSD spectra (upper Traces) in the previous figure. The calculated :2Z PSD does not match with the PSD of :1Z[1], probably because of its (over 20 dB) less significant amplitude. :Z1 was the dominant Force driving the bridge during testing.

Another method of calculating the force PSDs is to apply the **Transform | Spectrum** Command to the calculated force Time Waveforms in the [1] **Force Time Histories.BLK** Data Block. The following figure illustrates this result, using the same **Spectrum Averaging** parameters listed in the **Calculating PSDs** section. Note the improved match between the :2Z and :2Z[1] spectra.



PSDs calculated from the Force Time Waveforms.

### USING A SHAPE TABLE TO SYNTHESIZE FRFs

The FRF matrix of equation (1) can also be synthesized using a modal model contained in a *Shape Table*. FRFs can be synthesized using Shape Tables that contain either *Unit Modal Mass* (UMM) or *Residue* mode shapes.

When Forces are calculated using a Shape Table, the required FRFs are synthesized to match the frequency domain parameters of the Responses.

When *Residue* Mode Shapes are used, you can calculate Forces only at the *Reference DOFs* of the shapes. When *UMM* Mode Shapes are used, Forces can be calculated at any DOF of the mode shapes.

### An Example Using UMM Mode Shapes

We will repeat the prior two examples using a Shape Table with UMM Mode Shapes to synthesize the required FRFs.

- Close the **Z24 FRFs & Cohs.BLK** window.
- Minimize the **[1] Force PSDs.BLK[1]** window.
- Open the **Z24 Bridge 8-mode fit.SHP**, which contains a UMM modal model.

**Z24 Bridge 8-mode fit.SHP** contains 8 mode shapes obtained by curve-fitting the Z24 bridge FRFs. These mode shapes are in UMM format. UMM mode shapes contain *displacement* response units. However, we can use The **MIMO | Response** command to calculate Responses *with acceleration, velocity or displacement units*.

### Time Domain Force Calculation

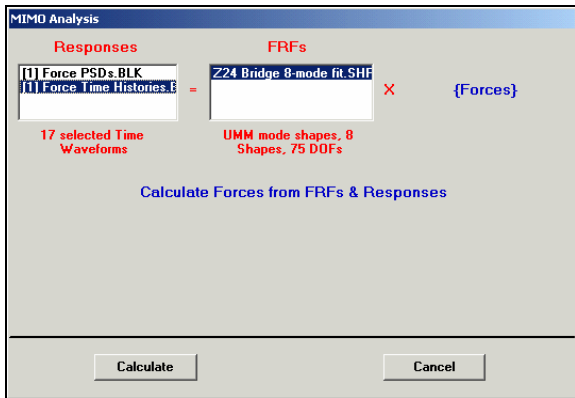
- Open the **[1] Force Time Histories.BLK** window.

In the **[1] Force Time Histories.BLK** window:

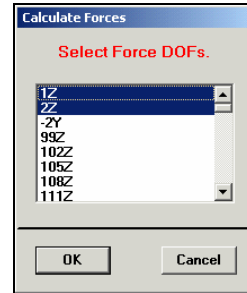
- Select the Response Traces **M#1** through **M#15**.



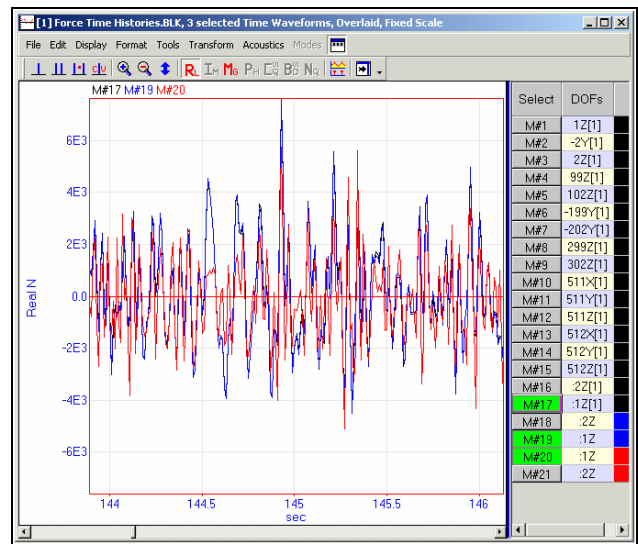
Execute: **Transform | MIMO | Forces**. The **MIMO Analysis** dialog will open.



- Verify that **Z24 Bridge 8-mode fit.SHP** is the source of **FRFs** and **[1] Force Time Histories.BLK** is the source of **Responses**. Then, click on **Calculate**. The **Calculate Forces** dialog will open.



- Hold the *Control* key down and *click* on **1Z** and **2Z** to select them as the desired force DOFs for calculating Forces.
- Press the **OK** button. When the calculation is completed, the **MIMO Calculations** dialog will open.
- *Click* on **OK**. The **Data Block Selection** dialog will open.
- Press the **Add To** button. The calculated Forces will be added to the **[1] Force Time Histories.BLK** Data Block as **M#20** and **M#21**.
- Select **M#20** and **M#21** and set their Trace Color to **red**.
- Compare the results in an overlay plot.

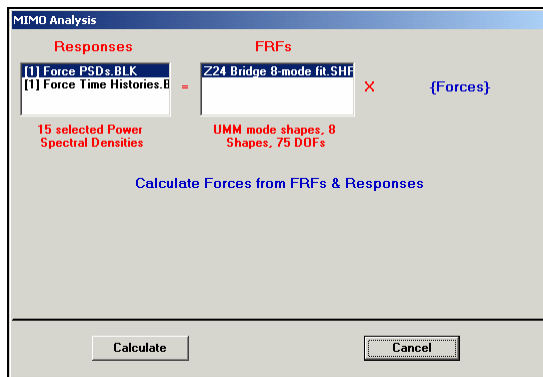


Shape-based, FRF-based and measured :1Z Forces.

### Force PSD Calculation

- Minimize the [1] Force Time Histories.BLK window.
- Open the [1] Force PSDs.BLK[1] window.
- Select the Response Traces M#1 through M#15.

Execute: **Transform | MIMO | Forces**. The **MIMO Analysis** dialog will open.



- Verify that **Z24 Bridge 8-mode fit.SHP** is the source of **FRFs** and that **[1] Force PSDs.BLK[1]** is the source of **Responses**. Click on **Calculate**. The **Calculate Forces** dialog will open.
- Proceed as in the prior **Time Waveform Calculation** example. Add the new PSDs as **red** Traces in the **[1] Force PSDs.BLK[1]** Spreadsheet.
- Compare the results in an overlay plot.



Shape-based, FRF-based and measured :1Z force PSDs.

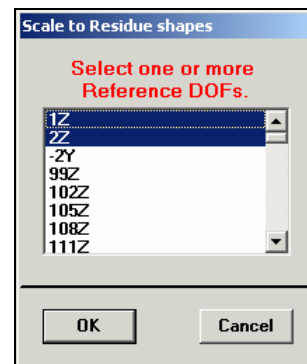
### Using Residue Mode Shapes

Residue mode shapes are normally created by saving shapes during curve fitting, but they can also be created by rescaling UMM mode shapes.

To calculate Forces using Residue Mode Shapes:

- Open the **Z24 Bridge 8-mode fit.SHP** file window.

Execute: **Tools | Scaling | Residues**. The **Scale To Residue Shapes** dialog will open.



- Select **Z1** and **Z2** as the Reference DOFs and click on **OK**.

The Shape Table is now converted to Residue mode shapes with **:1Z** and **:2Z** References. These Residues correspond to the two columns of the FRF matrix for force inputs at DOFs **1Z** and **2Z**.

- Repeat the **Transform | MIMO | Forces** calculations as done in the previous example. The results should be *identical* to those using the UMM Mode Shapes.

### SUMMARY

In this note, forces were calculated from measured responses for the Z24 bridge using both *FRFs* and *mode shapes* to represent the dynamic MIMO model. Forces were calculated both as Time Waveforms and PSD Spectra, and the results were compared. All of the comparisons showed good agreement between the measured forces and the forces calculated from the measured responses and the MIMO model.