



Choosing Response DOFs for a Modal Test

INTRODUCTION

Finite Element Analysis (FEA) can provide excellent guidance for conducting an experimental modal analysis. A well thought out FEA-based test plan reduces testing time and leads to better experimental results.

In this note we will use FEA mode shapes to *plan where responses should be measured* on a physical structure. We will answer important questions such as:

- What is the minimum number of Response accelerometers (or impact points) required to distinguish one mode shape from another?
- How many Response DOFs are required to make a satisfactory graphic display of these mode shapes?

HOW MANY RESPONSE DOFs ARE NEEDED?

Most FEMs contain far more DOFs than an experiment can match practically. A modal test will typically capture a much smaller number of responses. How many response points are required to validate the model by testing? How do you select them?

Three factors are important:

- The test DOFs should be a *subset* of the FEM DOFs, so that comparisons can be made directly, without the use of spatial interpolation.
- The test Response DOFs *must* contain the Reference DOF (or DOFs), so that a complete UMM modal model can be formed.
- Sufficient Response DOFs must be taken to allow *unambiguous verification* of every Mode Shape. (This can be a smaller number of points than is required to draw a detailed picture of each mode.)

Steps in the application note can be duplicated using **VT-550** Visual Modal Pro or any package that includes option **VES-350** Advanced Signal Processing.

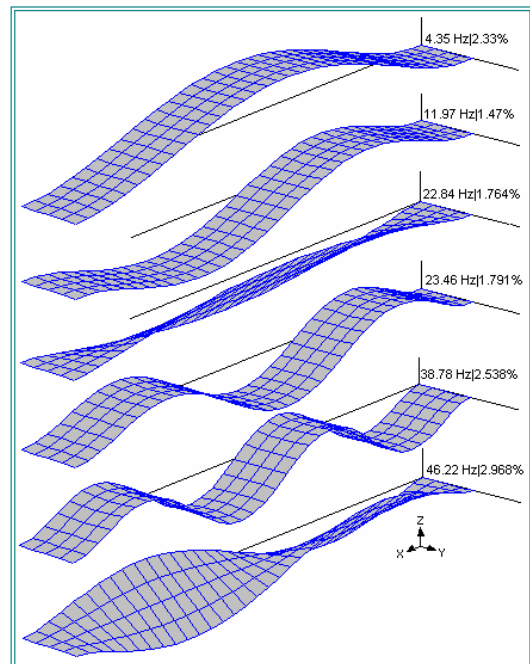
Clearly, Frequencies and Damping *could* be verified using only a single Response measurement. In principle, the FEM Mode Shapes *could* be verified by measuring as few as *one FRF per mode*. Such severe economy rarely proves practical. We will learn how to select the *minimum* number of DOFs needed *to test validate* an FEA model.

It is normally considered good practice to use sufficient test DOFs to characterize each Mode Shape with *reasonable fidelity for human viewing*. *Reasonable fidelity* is commonly accepted to be at about **10** points/cycle along any axis of sinusoid deformation.

THE FEM MODEL

The FEA modeled test article is a small-scale bridge model with a span of 27 feet and a width of 3 feet. Its ends are pinned to rigid attachment supports.

The finite element model contains six modes below 50 Hz and 175 Z-direction degrees-of-freedom (DOFs). All of the modes have modest damping of less than 3%.



Six FEA mode shapes containing 175 Z-Direction DOFs.

This model bridge was introduced in Application Note #24. In that note we determined the optimum excitation or Reference DOFs to be **40Z** for a *single reference* test and **65Z** and **88Z** for a *multiple reference* test.

Opening the Project

 Open **ME'scopeVES**.

 Execute: **File | Project | Open**.


- Select **Model Bridge** from the **More Examples /Application Notes/#24** subdirectory.

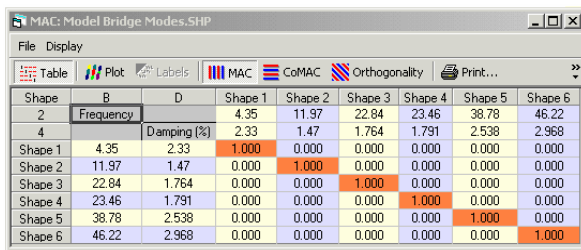
The **Model Bridge.STR** Structure and **Model Bridge Modes.SHP** Shape Table windows will open.

SELECTING THE FEWEST DOFs FOR A FEM VALIDATION TEST

Clearly, the **175** DOFs of the FEA mode shapes shown on page 1 are more than adequate to describe the six modes. However, we are often required to simply *test validate* an FEA using the smallest test possible. We need a scientific means of selecting a small sub-set of these DOFs that will still accurately represent the *shapes*. This is provided by the *Modal Assurance Criterion* (MAC).

- Close the **Model Bridge.STR** window.

 Execute: **Display | MAC** in the **Model Bridge Modes.SHP** window. The **MAC** Table will open.



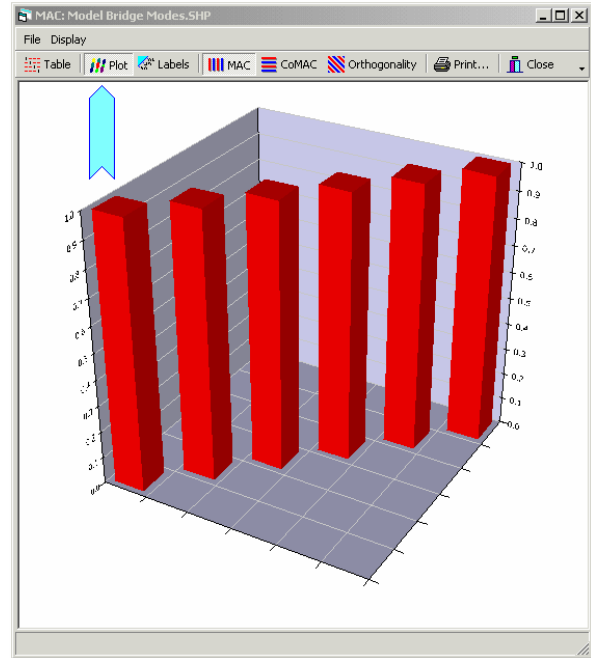
Shape	B	D	Shape 1	Shape 2	Shape 3	Shape 4	Shape 5	Shape 6
2	Frequency		4.35	11.97	22.84	23.46	38.78	46.22
4	Damping (%)		2.33	1.47	1.764	1.791	2.538	2.968
Shape 1	4.35	2.33	1.000	0.000	0.000	0.000	0.000	0.000
Shape 2	11.97	1.47	0.000	1.000	0.000	0.000	0.000	0.000
Shape 3	22.84	1.764	0.000	0.000	1.000	0.000	0.000	0.000
Shape 4	23.46	1.791	0.000	0.000	0.000	1.000	0.000	0.000
Shape 5	38.78	2.538	0.000	0.000	0.000	0.000	1.000	0.000
Shape 6	46.22	2.968	0.000	0.000	0.000	0.000	0.000	1.000

MAC matrix using all 175 DOFs of FEA mode shapes.

Note that the MAC matrix is a **6-by-6 identity matrix** with **1's** on the diagonal (orange highlight) and **0's** in all of the off-diagonal locations. These MAC coefficients compare the *shapes* in the model with on another. Each row and each column represent one Mode Shape and the coefficient at each intersection expresses the similarity of a row and column mode. **1** indicates a perfect match, **0** indicates no similarity. A value of **0.9** indicates strong similarity between the two shapes.

Since we are comparing the six modes of an FEA, we expect the MAC to be a perfect identity matrix. Each of the mode shapes is *linearly independent* of the others, as the shapes are known to be *orthogonal* with respect to the models's mass matrix.

- Press the **Plot** button in the **MAC** window. The Table will be replaced by a **MAC Plot**, as shown below.

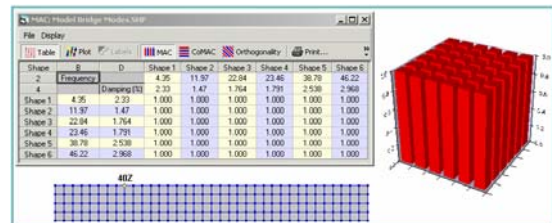


Plot of MAC matrix for all 175 DOFs.

- To restore the Table, press the **Table** button.

The preceding **MAC** figures used all **175** DOFs of the FEA mode shapes. Now we will investigate using a much smaller number of measurement sites.

- **Click on M#40 (40Z)** to Select it in the **Model Bridge Modes.SHP** Spreadsheet. Note the immediate effect on the **MAC** matrix.

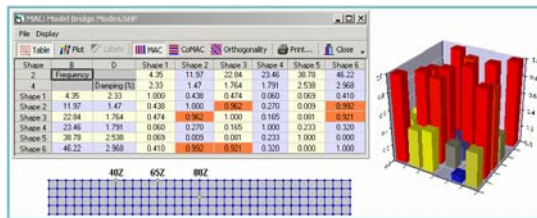


MAC for DOF 40Z only.

With *any* single DOF selected, the **MAC** matrix has **1**'s in every location. It is clear that *shapes* cannot be discriminated using only 1 DOF.

Note that **40Z** is the optimum Reference DOF for a *single reference* test. We would like to build a test model that can serve to validate the FEA using either *single reference* or *multiple reference* testing. Therefore, the model must also include DOFs **65Z** and **88Z**.

- Select measurements **M#65 (65Z)** and **M#88 (88Z)** in the **Model Bridge Modes.SHP** Spreadsheet. Note the immediate effect on the **MAC** matrix, shown below.

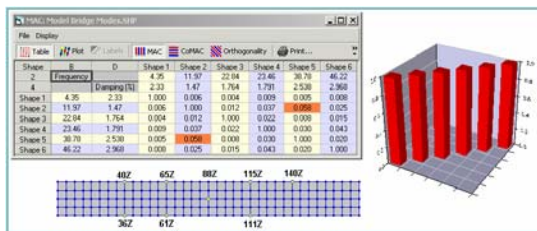


MAC for Reference DOFs **40Z**, **65Z** and **88Z** only.

With the three Reference DOFs selected, the **MAC** matrix has far fewer significant elements. All of the diagonal elements have a value of **1**. However, six off-diagonal elements (highlighted in orange) still have values greater **0.9**, indicating that this set of three DOFs cannot discriminate between mode shapes **2**, **3** and **6**.

We need to add additional DOFs to improve the shape selectivity. As a criterion for success, we will demand that *all off-diagonal MAC elements have a value of 0.1 or less*. The objective is to *add as few DOFs as possible*, to achieve this goal. One possible solution is the addition of DOFs **36Z**, **61Z**, **11Z**, **115 Z** and **140Z**.

- Select additional DOFs **36Z**, **61Z**, **11Z**, **115 Z** and **140Z** in the **Model Bridge Modes.SHP** Spreadsheet.

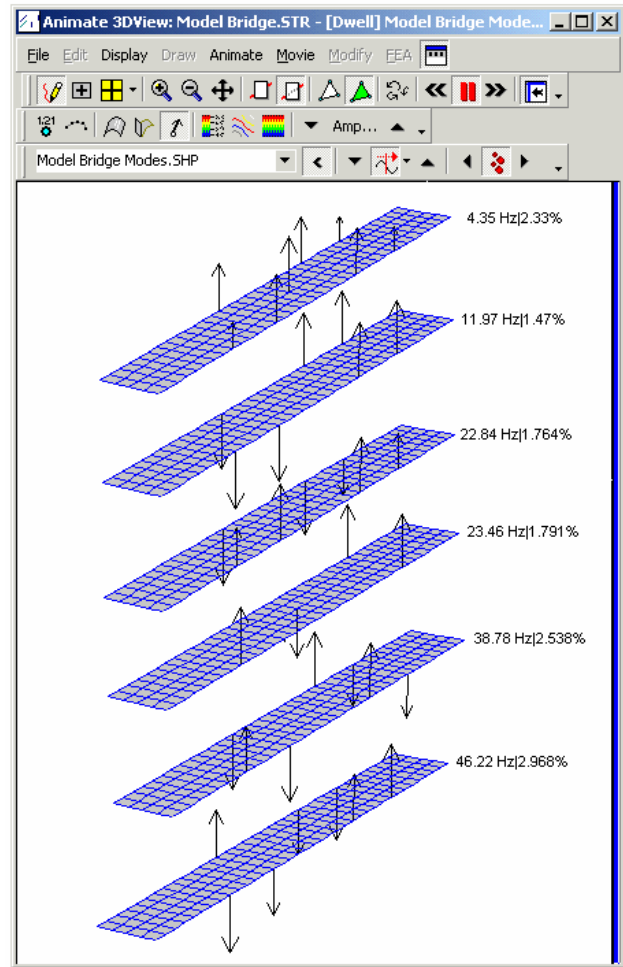


MAC using eight DOFs including the three References.

Note that our criterion is solidly met. The worst-case off-diagonal elements have a value **0.058**.

This MAC solution indicates that *these eight DOFs are sufficient to validate the FEA model's six mode shapes*. Either a *single reference* test measuring **8** FRFs against **:40Z** or a (two-shaker) *multiple reference* test measuring **16** FRFs against **:65Z** and **:88Z** may be employed.

The “correctness” of each *test measured* mode shape may be evaluated by calculating a Cross MAC between the measured and FEA mode shapes. However, such a “minimized” test will not provide *reasonable fidelity* for human viewing as illustrated below. Eight DOFs are simply insufficient to capture the graphic nature of each mode shape in human interpretable format.

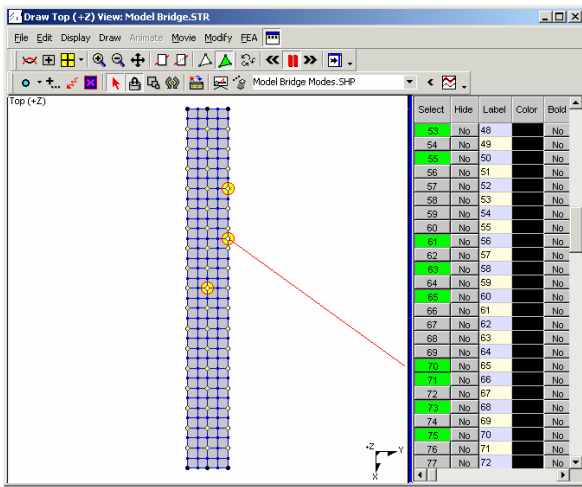


Mode Shapes represented by eight DOFs.

CHOOSING RESPONSE DOFs FOR ANIMATION

Inspection of this note’s first figure discloses that sine-like deformation occurs along the X-axis of the bridge. The highest “spatial frequency” is two sine cycles along the span. This occurs in the **38.78 Hz** 5th mode. Hence we want about **20** equally-spaced points in the X-

direction. Along the Y-axis, the deformations appear to be straight lines. Three equally-spaced points along the Y-direction will be sufficient to verify this shape.



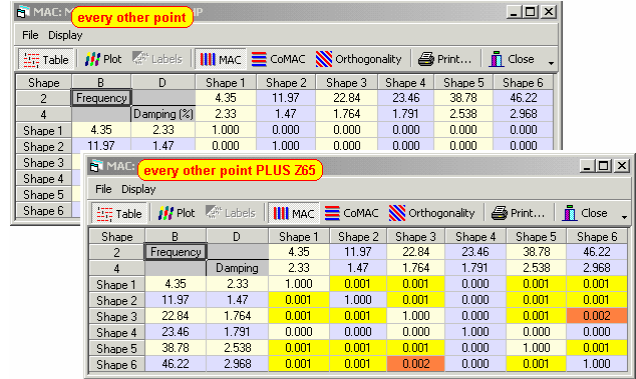
58 Point model provides reasonable fidelity for 6 modes

As shown by the figure above, selecting every other point from our (very small) FEM will result in a grid with 19 points along the X-axis and 3 along the Y-axis. This agrees well with our reasonable fidelity guidelines. Of these 57 points, 6 are fixed boundary points that do not move for which measurements will not be required. Since our FEM only contained Z-direction DOFs, our test model is reduced to 51 DOFs where Responses must be measured.

However, our “decimated” model must contain the DOFs 40Z, 65Z and 88Z, to be equally suitable for use in a Single Reference test (using 40Z as the shaker DOF) or a Multiple Reference test (applying the shakers to DOFs 65Z and 88Z). This requires adding 65Z as a 58th point (52nd DOF).

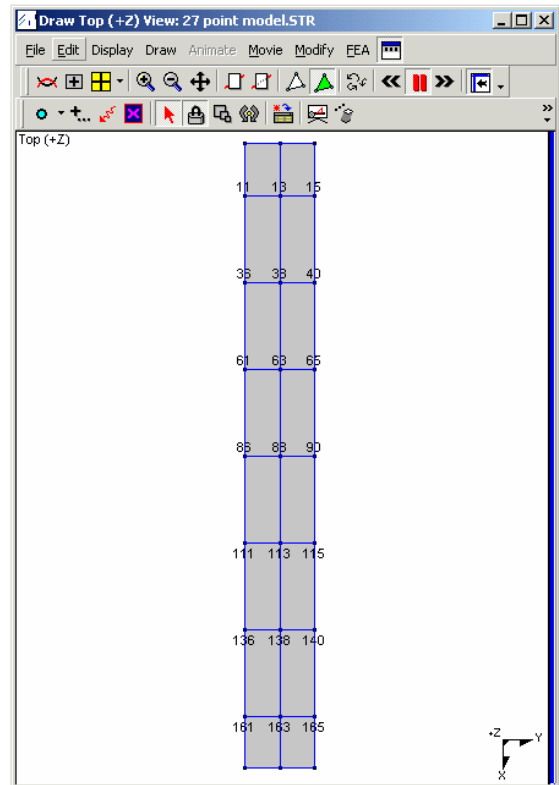
Can we verify that this model will provide unambiguous verification of each Mode Shape? Yes! By using the Modal Assurance Criterion (MAC) and a subset of the DOFs in the FEM Shape Table, we can verify that the model has sufficient spatial resolution to identify each shape uniquely.

As shown in the following figure, the MAC matrix for the (symmetrically arrayed) 51 DOFs is a perfect identity matrix. When we add 65Z, the computational symmetry is broken and some small off-diagonal terms result. The largest of these is 0.002 (shown in orange) and 18 terms of .001 are also noted (in yellow). These are very acceptable off-diagonal terms.



MAC matrix for 58 point, 52 DOF test model.

We will prove the necessity of the spatial sampling provided by the 58 Point, 52 DOF model by comparing it with a smaller model. The figure below illustrates a 27 Point model including 21 DOFs. This model only provides 9 points along the X-axis, less than half of those required by our “rule-of-thumb”.

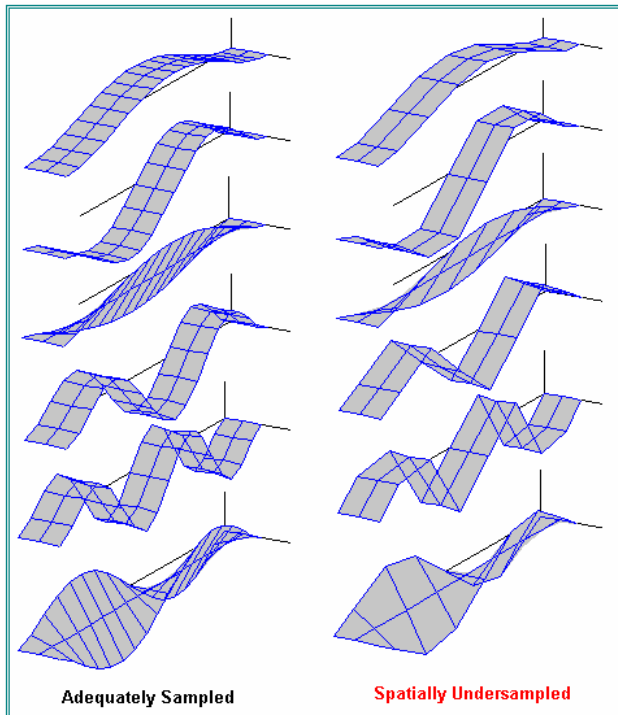


27 Point, 21 DOF structural model.

To effect this comparison:

- Execute: **File | Add** and select the **58 point model.STR** and **27 point model.STR** files from the **More Examples /Application Notes/#25** subdirectory.

- Following the instructions in the **Animating Mode Shapes** section of Application Note #24, animate the **Model Bridge Modes.SHP** shapes using **58 point model.STR** and then **27 point model.STR** and then again using **Model Bridge.STR**.



Comparison of FEA mode shapes using 58 Point (left) and 27 Point (right) Structure models.

As illustrated by the figure above, the mode shapes animated using the **58 point model.STR** convey all of the information provided by the original **Model Bridge.STR** model. Animation using **27 point model.STR** as the Structure, results in some confusion about the mode shapes, particularly for modes **4** and **5**.

SUMMARY

By following the instructions in this note you have:

- Used the Modal Assurance Criterion to determine a minimum number of Response DOFs for an FEA validation test.
- Used a simple 10 point-per-cycle “rule of thumb” to select the minimum necessary Response DOFs for animated displays with *reasonable fidelity for human viewing*.

Related Studies

In Application Note #26, you will *simulate acquisition* of measurements from the test article, using the **8** DOF and **52** DOF models derived here. In that effort, you will simulate both *single reference* and *multiple reference* test using the optimum Reference DOFs determined in Application Note #24.