

# Display Operating Deflection Shapes from Nonstationary Data

Håvard Vold, Vold Solutions, Inc., Cincinnati, Ohio

Brian Schwarz and Mark Richardson, Vibrant Technology, Inc., Jamestown, California

**In this article, a new methodology is presented for post-processing nonstationary operating data as a prerequisite for displaying operating deflection shapes on a 3D spatial model of a test machine or structure. The traditional 'transmissibility' measurement, which is a measure of each response normalized by a reference response, is discussed. Then, two new post-processing methods associated with two new types of measurements (the ODS FRF and the ODS Order Track) are introduced and their use is illustrated with data typical of realistic testing situations.**

Vibration problems in structures or operating machinery often involve the excitation of structural resonances or modes of vibration. Many types of machinery and equipment can encounter severe resonance related vibration problems during operation. In order to diagnose these problems, an animated display of the machine or structure's operating deflection shapes is often very helpful. In most cases, structural responses at or near a resonant (modal) frequency are 'dominated' by the mode, and the ODS closely approximates the mode shape.

Modal testing (performing a modal survey) is usually done under controlled stationary (non-time varying) conditions, using one or more exciters. Furthermore, the excitation forces and their corresponding responses are simultaneously measured. In many cases, especially with operating equipment, the measurement signals may be nonstationary (time varying) and the excitation forces are not measured. For these cases, different post-processing is required in order to display ODSs from a set of measurements.

Two different types of nonstationary signals are addressed in this article (in both cases the excitation forces are not measured): variable force level – excitation *force levels change* during the measurement time period; variable rotational speed – the excitation *forces are cyclical* and are related to a rotational speed in the machine, which changes during the measurement time period.

## Requirements for an ODS

In general, an Operating Deflection Shape (ODS) is *any forced motion for two or more* DOFs (Degrees of Freedom – points and directions) on a machine or structure. An ODS, therefore, defines the *relative motion* between two or more DOFs on a structure. An ODS can be defined for a specific frequency or for a moment in time.<sup>2</sup>

This definition of a 'shape' requires that all *measured responses have correct magnitudes and phases relative to one another*. In order to ensure that a set of vibration measurements taken from two or more DOFs has the correct relative magnitudes and phases, two methods of measurement can be used: simultaneous method – simultaneously acquire all responses together; measurement set method – simultaneously acquire some of the responses, plus a *reference* (fixed) response. This is called a *measurement set* (an entire test, then, consists of acquiring two or more measurement sets).

Most machinery maintenance and product development organizations cannot afford large multi-channel systems that simultaneously acquire all channels of data at once. As a result,

most vibration measurements are made with equipment that can only acquire a few channels of data at a time. For these reasons, we will focus on the measurement set method as the more commonly used acquisition method.

With the measurement set method, a reference (fixed) response must be measured with each measurement set in order to preserve the relative phase among all responses in all measurement sets. If the phase of the reference response is subtracted from the phase of each response in its measurement set, the responses from all measurement sets will have the correct phase relative to each other. This is the same as using the phase of the Cross Power Spectrum (CPS) between each response and the reference response.

Using the measurement set method, the time period required to obtain a set of measurements can be substantial and the machine or structure could physically change during this time period. Furthermore, the excitation forces could change. Either of these cases will result in nonstationary measurement signals.

## Nonstationary Structural Properties

Nonstationary vibration signals will result if the physical properties of the structure or machine (its mass, stiffness and damping) change during the measurement time period. For example, fluids moving within the system can cause mass changes. Temperature changes can cause changes in material stiffnesses. Damping characteristics can also change during the course of a prolonged test.

We can determine whether or not a structure is stationary by applying the following definition to successive measurements: stationary measurement – a vibration signal is stationary if its Auto Power Spectrum (APS) does not change from measurement to measurement. In other words, if two or more APS measurements are made from the same response DOF (point and direction) and overlaid on one another, and if they are all essentially the same waveform (with the exception of small amounts of measurement noise), then the structure can be said to be stationary.

Modes (resonances) are inherent properties of a structure. They will only change if the physical properties or boundary conditions of the structure change. Modes are manifested as 'peaks' in any response APS measurement. If the peaks in two successive APS measurements are at the same frequencies when the two APS are overlaid, then we can conclude that the modes (and hence the physical properties) of the structure are stationary.

If only the amplitudes or levels of the response signals are different when successive APSs are overlaid, then special post-processing must be applied to the signals to recover Operating Deflection Shapes. In this case, it is assumed that the unmeasured excitation forces are nonstationary. Recovery of ODSs when excitation levels change is one of the methods presented here.

## Order Related Excitation

Another condition under which vibration measurement signals are nonstationary almost always occurs with rotating equipment. Rotating machines generate internal cyclic excitation forces that are related to the rotational speed(s) of one or more rotating components. These forces are said to be *order related*.

<sup>2</sup>Based on a paper presented at IMAC-XVIII, the 18<sup>th</sup> International Modal Analysis Conference, San Antonio, TX, Feb. 2000.

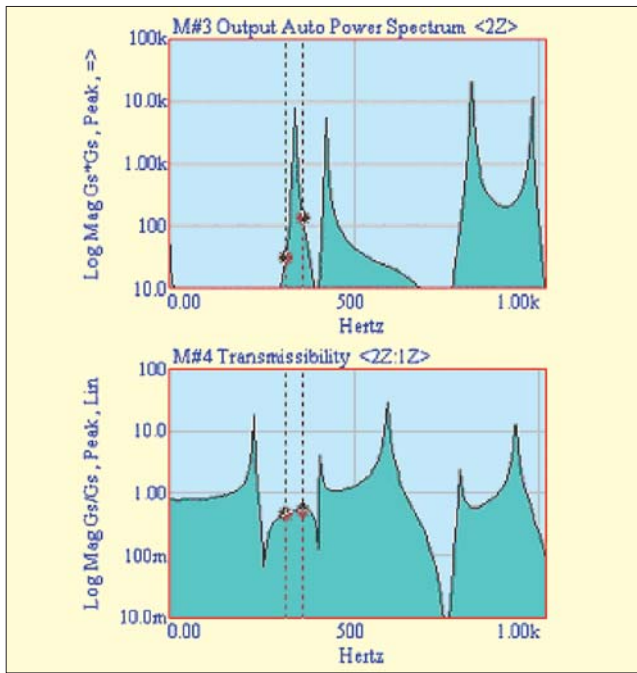


Figure 1. APS and transmissibility.

Order related excitation results in forced responses with frequencies that are multiples of a rotational speed of the machine. These cyclic forces are also manifested as ‘peaks’ in any vibration response APS measurement taken from the machine. So, if the excitation is order related and resonances are also being excited, an APS measurement will contain peaks due to both the excitation and the resonance response.

In order processing, resonance peaks are assumed to be stationary, while the order related forced response peaks are nonstationary. If the rotational speed on the machine can also be measured together with vibration signals, the vibration signals can be specially processed using order tracking methods. Then, order tracked ODSs can be displayed from multiple measurement sets of operating data. This is the second post-processing method introduced in this article. The point is, for a given rotational speed and harmonic, the amplitude and phase responses are repeatable between runs, excepting for transients, which pose no problem at reasonable slew rates.

Reference 3 describes the time variant Fourier transform order tracking algorithm used in VSI Rotate 2™, the program used for the example later in this article. References 4 and 5 are good tutorial/survey papers of the order tracking technology, and References 5 and 6 describe the latest innovations in high performance order tracking algorithms.

### Transmissibility

Transmissibility is the traditional way of making a frequency response measurement when the excitation force(s) cannot be measured. It is calculated in the same way as the Frequency Response Function (FRF) or Transfer Function and is a standard measurement made by all 2-channel FFT analyzers. Whereas the FRF is the ratio of response divided by force, *transmissibility is the ratio of a response divided by a reference response.*

Transmissibility is a *ratio of two responses.* Therefore, if the excitation force level varies from one measurement to the next, it is assumed that its effect on both responses is the same, so its effect will be “canceled out” in the transmissibility measurement.

ODSs can be displayed from a set of two or more transmissibility(s). A set of transmissibility(s) can be measured one at a time using a 2-channel analyzer, where each measurement uses a different (roving) response divided by the reference response from the same fixed DOF.

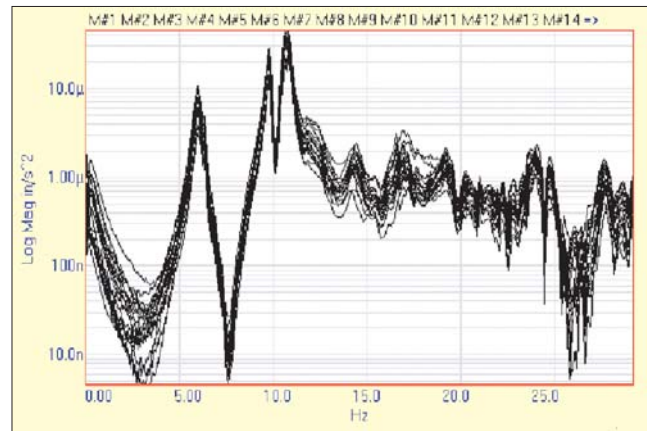


Figure 2. Twenty reference APSs overlaid.

**Difficulty with Transmissibility(s).** The only difficulty with a transmissibility measurement is that peaks in the measurement are not evidence of resonances. Rather, resonant frequencies are located at “flat spots.” Therefore, if a set of transmissibility(s) is used for displaying ODSs, at least one APS measurement must also be used for locating resonance peaks.

Figure 1 shows a response APS plotted above a transmissibility. At the frequency of the resonance peak inside the cursor band (shown with dashed lines) in the APS, the transmissibility has a flat spot, not a peak. Moreover, the peaks in the transmissibility do not correspond to resonances but are merely the result of the division of the roving response by the reference signal at frequencies where it is relatively small.

### ODS Frequency Response Functions

A new type of measurement has been defined<sup>1</sup> from which ODSs can be obtained in a more straightforward manner. The ODS FRF is a complex valued frequency domain function: *the ODS FRF is formed by combining the APS of a roving response with the phase of the CPS between the roving response and the reference response.*

A set of ODS FRFs can be measured in the same way as a set of transmissibility(s), using a 2-channel analyzer. But, instead of being a ratio of responses, the ODS FRF contains the correct magnitude of response (the APS of the roving response), and the correct phase relative to the reference response. More importantly, an ODS FRF has *peaks at resonances*, so it is much easier to display ODSs from a set of ODS FRFs and observe mode shapes at resonant frequencies.

**Correction of ODS FRF Magnitudes.** Whereas the transmissibility automatically takes care of the effects of nonstationary signals due to the variable force level case, the ODS FRF does not. Therefore the magnitudes of a set of ODS FRFs must be corrected to account for changes in the excitation level between measurement sets. To correct for differing excitation levels, the magnitude of each ODS FRF in a measurement set *i* is multiplied by the scale factor *i*, where

$$\text{Scale Factor}(i) = \frac{\sum_{i=1}^{\text{No. of Meas. Sets}} \text{ARM}(i)}{(\text{No. of Meas. Sets}) \times \text{ARM}(i)} \quad (1)$$

where:

ARM(*i*) = average value of the reference response APS for measurement set *i*.

This scale factor corrects each of the ODS FRF magnitudes according to the *average level of all of the reference response signals* (this average value can be calculated for any desired range of frequency samples).

In order to form a set of properly scaled ODS FRFs, the following functions are needed: the APS of each roving response, the APS of each reference response, and the XPS between each roving and reference response pair. These three functions are a typical result of trispectrum (or cross channel) averaging

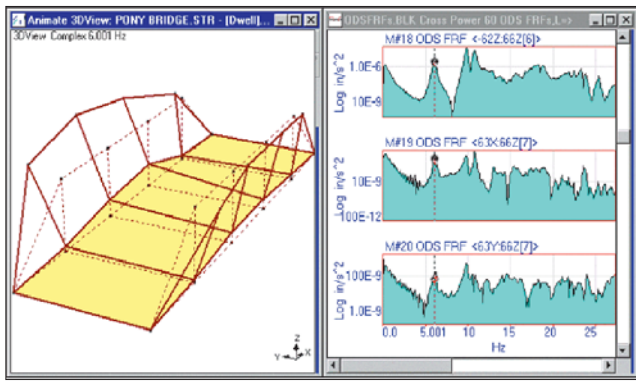


Figure 3. Mode shape of the truss bridge.

that is implemented in any modern 2-channel FFT analyzer.

**ODS FRF Example.** In this example, a truss bridge was tested using a 4-channel analyzer and impact excitation. Because the peak loads due to impacting were too great to measure with a load cell, a reference accelerometer was used instead. A roving triaxial accelerometer was used to measure 3D responses at 20 points on the bridge. Each measurement set consisted of APSs for three roving responses, an APS for the reference response, and three CPSs (between each roving and the reference response).

Figure 2 shows all 20 reference APSs overlaid on one another. It is clear from this overlay that the impacting force level is different among the 20 measurement sets. These data are clearly a *variable force level* case, requiring that the ODS FRF magnitudes be rescaled using the formula in Equation (1). Figure 3 shows a mode shape of the bridge obtained by displaying the ODS at one of the resonance peaks in the ODS FRF data.

### ODS Order Tracks

Most vibration measurements taken from an operating machine with rotating components are nonstationary according to the variable rotational speed case. That is, the unmeasured excitation forces are cyclic and are directly related to the rotational speed of the machine, which may change throughout the testing period.

If the *rotational speed of the machine is simultaneously acquired* with the vibration response signals, then the sampled time domain signals can be post-processed to yield a new set of order tracked functions. These order tracks are complex valued (having both *magnitude and phase*), and are functions of either time or rotational speed.

For ODS order tracks, each measurement set consists of one or more roving responses, a reference response and a tachometer signal. ODS order tracking is done in several steps:

1. Multiple measurement sets are acquired and saved in mass storage.
2. The tachometer signal is processed for each measurement set to obtain the machine speed as a *smooth continuous function* of time.
3. All of the response signals in a measurement set are “order filtered” using a band pass filter that varies with the speed of the machine.<sup>3</sup>
4. The phase of each reference response is *subtracted* from the phase of each roving response in its measurement set.
5. The magnitude of each roving response in each measurement set is multiplied by the scale factor defined in Equation (1).
6. The modified roving responses (magnitudes and phases) from all measurement sets are used to display ODSs.

Step 4 makes the phases of all roving responses in a measurement set relative to the reference response. Since the same reference response DOF is used for all measurement sets, all of the roving responses will have the correct phase relative to one another.

Step 5 corrects all roving response magnitudes accord-

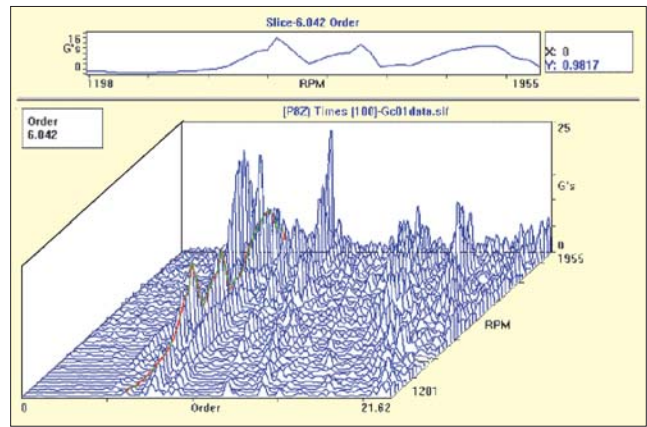


Figure 4. Waterfall showing sixth order exciting resonances.

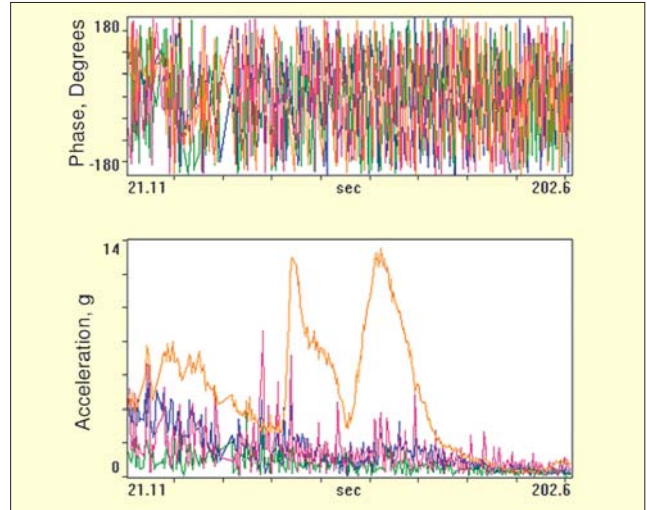


Figure 5. Order tracks of sixth order from measurement set [1].

ing to the average level of all of the reference responses, as explained earlier. This correction accounts for any occurrences of the variable force level case between the measurement sets.

**ODS Order Track Example.** In this example, vibration measurements were made on a gearbox cover on a rotating machine during machine run-ups. The cover was experiencing premature failures, so the testing was done to determine if any resonances were being excited by orders of the machine.

A 12-channel system was used for data acquisition. There were 33 test points on the cover. Three triaxial accelerometers were used to acquire roving response data, three points at a time. A uniaxial accelerometer was used to acquire the reference response. A total of 11 acquisition channels was used for each measurement set, nine for the roving responses, one for the reference response, and one for the tachometer signal.

A total of 12 measurement sets was acquired. Ten sets had nine roving responses in them, one set had six roving responses, and the 12<sup>th</sup> set had only three roving responses, giving a total of 99 roving responses from all measurement sets. When displayed as an ODS, these data provided 3D motion of the cover at all 33 test points.

The VSI Rotate 2 software package was used to perform order tracking on the 12 measurement sets. Figure 4 shows a waterfall plot of a typical roving response prior to order tracking. This waterfall is simply a series of APSs. Notice that each APS (vertical axis) is a function of Order (horizontal-axis) and that each APS corresponds to an RPM (skewed-axis).

The waterfall indicates that the sixth order is exciting one or more resonances, evidenced by the peaks in the ‘slice’ for

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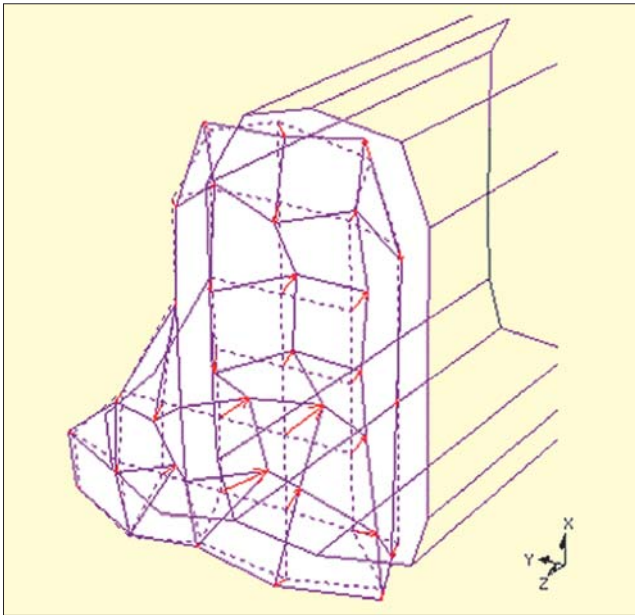


Figure 6. ODS of gearbox at 1600 RPM.

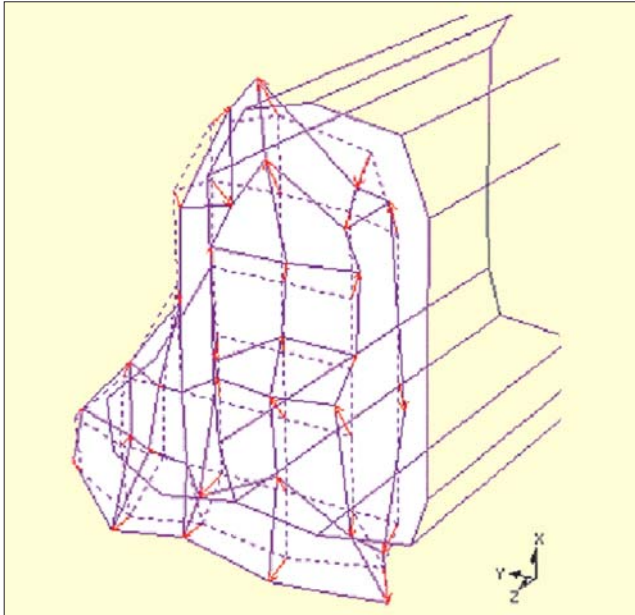


Figure 7. ODS of gearbox at 1800 RPM.

the sixth order, displayed above the waterfall. This preliminary analysis led us to compute order tracks for the sixth order. Figure 5 shows a typical set of order tracks for the sixth order, for one measurement set.

After order tracking was performed on all 12 measurement sets, they were exported from VSI Rotate 2 to ME'scopeVES™. The final two data processing steps (steps 5 and 6 described earlier) were performed in ME'scopeVES. This yielded a set of ODS order tracks, one for each roving response on the gearbox cover.

Finally, each of the 99 roving response ODS order tracks was assigned to its appropriate DOF of a 3D model of the machine, and the ODSs were displayed in animation. Figure 6 shows the ODS being excited by the sixth order at a machine speed of 1600 RPM. Notice that the predominant motion is normal to the plane of the gearbox cover. Figure 7 shows the ODS of the gearbox at 1800 RPM. Notice that the predominant motion is now in the plane of the cover. This was an unexpected result, which led engineers to investigate the possibility of failure due to excessive in-plane motion, rather than out-of-plane motion.

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## Conclusions

We have introduced two new post-processing techniques that enable the display of operating deflection shapes from data that are measured under nonstationary conditions. A valid ODS must have components or DOFs with *correct magnitudes and phases relative to one another*. This is guaranteed if all response channels of data are acquired simultaneously. Unfortunately, most testing organizations don't have the resources to acquire data in this manner. Rather, data can only be acquired a few channels at a time in several measurement sets.

Two nonstationary test cases were defined that commonly occur when testing structures and operating equipment. Both cases involve variations in the excitation forces that also are not measured. It was shown that both magnitude and phase corrections can be made to the response data if a reference response is acquired with each measurement set.

Finally, if the primary excitation force is a function of the speed of a rotating component of a machine (i.e., it is order related), then a tachometer signal must also be acquired with each measurement set. Order tracking can be performed on each measurement set using the tach signal and order tracking post-processing software. Following magnitude and phase corrections, order tracked responses can be displayed as ODSs on a 3D model of the test machine or structure.

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